Origami Tessellations *for Everyone*

30+ Original Designs
by Ilan Garibi

Hand-Drawn Diagrams by Franziska Schwarz

Origami Tessellations for Everyone

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Cover image: verso view of Big and Small Cubes tessellation (page 67) from red elephant hide paper.

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To my wife, who shares my path in life

Acknowledgments

This book is a summary of the last ten years, since I took my first steps to become an origami artist. For 25 years I was a soldier, and this second career—becoming an artist, of all things—was a major surprise to me and to all who know me.

My major area of interest is tessellations, although I do have some modulars and figurative models in my portfolio. I adore the simple logic they follow, and the geometric aesthetic they convey.

I was delighted to learn that origami is not only about folding paper, but it is also about the people and communities. I met many people, and learned from each, a little or a lot. This journey had many turns, ups and downs, but all were around the same subject—origami.

This book would not be the same without the hand-drawn diagrams of **Franziska W. Schwarz**, an artist and an origamist. For more than three years—at in-person meetings (at origami conventions) and on Skype—she has labored to transform my rather plain diagrams into works of art. This is her book, just like it is mine!

Sherry Gerstein, my editor, raised this book to a professional level. With keen eyes and the full understanding of the common reader, she made sure this book will indeed fit everyone, beginners as well as practiced folders.

Elina Gor, my graphic designer, is a magician! With skillful hands and eyes, she beautified the book to a level beyond my expectations.

Many thanks to **Dasa Severova**, for walking me down the road of publishing with this book. Your tips and help have been invaluable!

My group of test folders—**Martin Rohrmann, Robin Scholz, Yara Yagi** and **Michał Kosmulski** (who also designed the only model in this book which is not mine)—showed me friendship and support, making this book a better one!

—Ilan Garibi

According to my mom, I started folding around the age of six during Christmas. If you come from Germany, the chances are quite high that the first thing you will ever fold is a Christmas star made of sheets of transparent colored paper. So paper folding at Christmas became a tradition for me. When I was 12, my mom and I were visiting a friend who had an astonishing paper object hanging from her chandelier. That very day I'd also learned my first Sonobe module and had already folded 52 of them! From that moment on, folding paper became an important part of my life. It appeals to my artistic side, but because of its strong affinity with mathematics, it appeals to my scientific side as well. To my great delight, my mother appreciates both aspects as well. She has been my biggest supporter and fan of all my achievements, either in arts or in science.

It was my mother who went with me to my first origami convention when I was in my twenties. I had just started my masters of science in nutrition and my time would have been much better employed by studying molecular formulas. But this origami convention was like stepping into another universe. Meeting all those amazing people and making new friends opened up a whole different world to me. I started attending other European conventions, and that is when I met Ilan Garibi. We met up again at an origami convention in Italy, and we became fast friends. Once he saw that I could draw origami diagrams, it was as if he'd found the missing piece to a puzzle he'd been working on. It is a little odd to think that those drawings were the start of an amazing collaboration, what you now hold in your hands as a finished book. I hope you enjoy it, this product of a passion that initially started at the age of six, was nurtured by a loving mom and supported by many incredible friends since then.

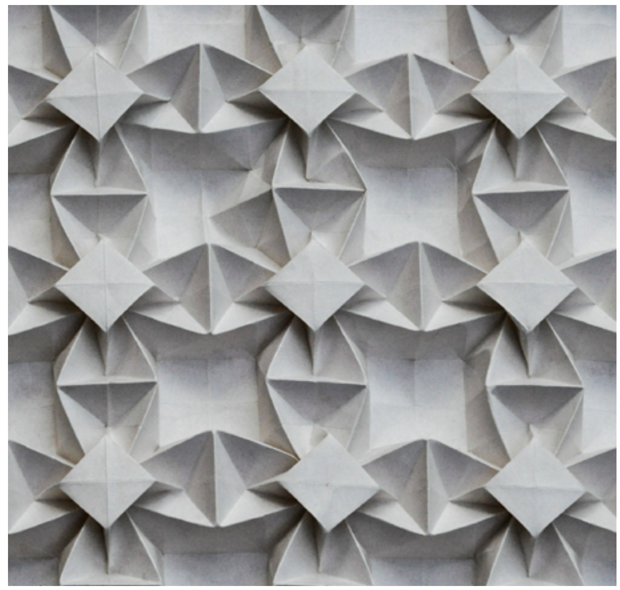
—Franziska Schwarz

Contents

Introduction

Methods for manipulating a fabric surface into a repetitive pattern have been known for centuries. Ruffles, pleats, and smocking are all well-established techniques in fashion, but it takes a considerable amount of costly materials (like the fabric), not to mention the know-how and experience to do it well. Luckily, you can manipulate paper (a much less expensive material!) in a similar way—and all you need is your fingers.

This may be the reason for the increasing interest and popularity of this origami genre in the international community.



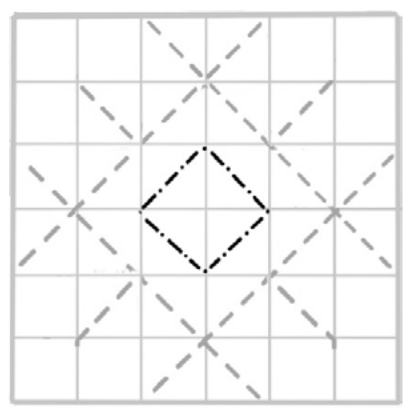
Templar Garden tessellation—the molecules expands identically in all directions.

What is an Origami Tessellation?

That's a good question.

Tessellation is a mosaic. In other words, it is a covering of an infinite geometric plane with repetitive geometric shapes, without gaps or overlaps. In origami, it is the name we give geometric origami folded with a single sheet of paper. There are many categories and subcategories—corrugations, curved tessellations, self-similarity folds, to name a few.

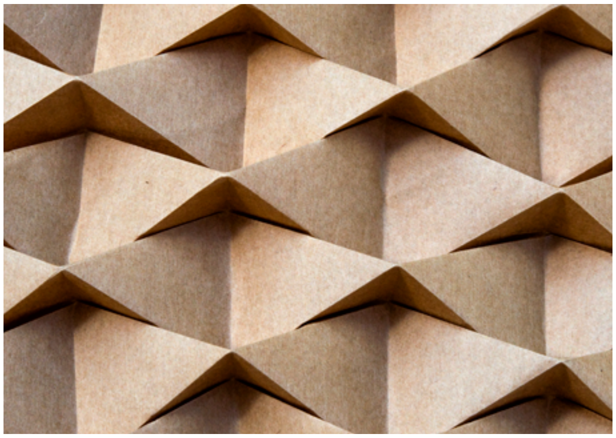
The core property of all tessellations is that of a mosaic, the repetitive pattern made by folding a single sheet of paper. A fold (or a group of folds) is repeated again and again, and in all directions. In this book, this grouping is called a **molecule**.



Templar Garden molecule crease pattern.

Some tessellations are based on a square grid (above) while others use a grid made of equilateral triangles that form hexagons.

A molecule can be identical on all four sides, like the Templar Garden molecule, or have a different pattern on the top and bottom than what's on the left and right sides, as shown below.



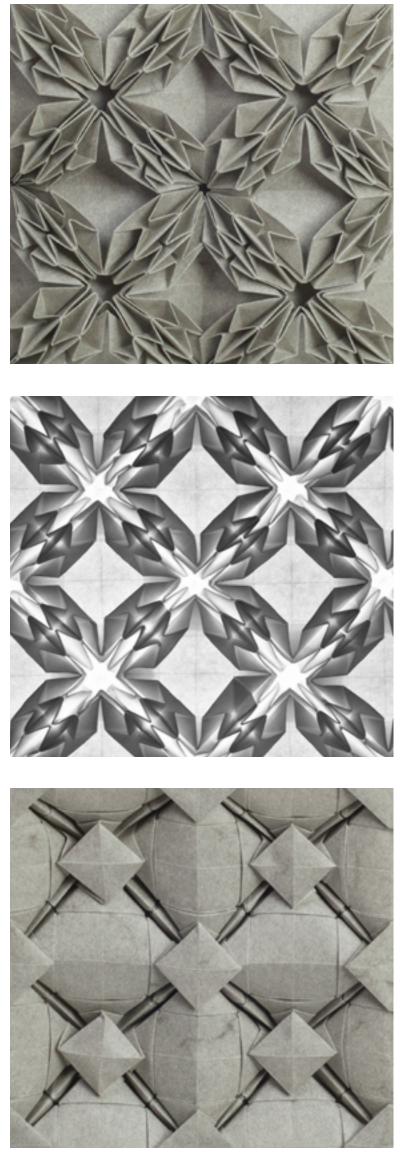
Diamonds corrugation.

Types of Tessellations

A **standard tessellation** is a mosaic, a surface covered without gaps or overlaps in the pattern. It can be flat or have dimensional features; it can be based on a square or a hexagon (AKA triangle) grid.

While the pattern itself doesn't overlap, the paper may have several overlapping layers. The surface of the final model will always have an odd number of layers (one, three, five or even more) throughout. This is because whenever the paper is first folded this way, it must be folded the other way, too, to allow continuity. This variation in the number of layers can be shown to great effect when you backlight your model. One layer is somewhat transparent; additional layers are progressively more opaque. For that reason, one tessellation can give you three distinct impressions: the first side, the other side, and either side when backlit.

Corrugation is a surface that has been formed into ridges, wrinkles, or furrows. This type of tessellation has no overlapping layers. The entire surface of the paper is visible to the eye, and the pattern is usually in the form of waves.

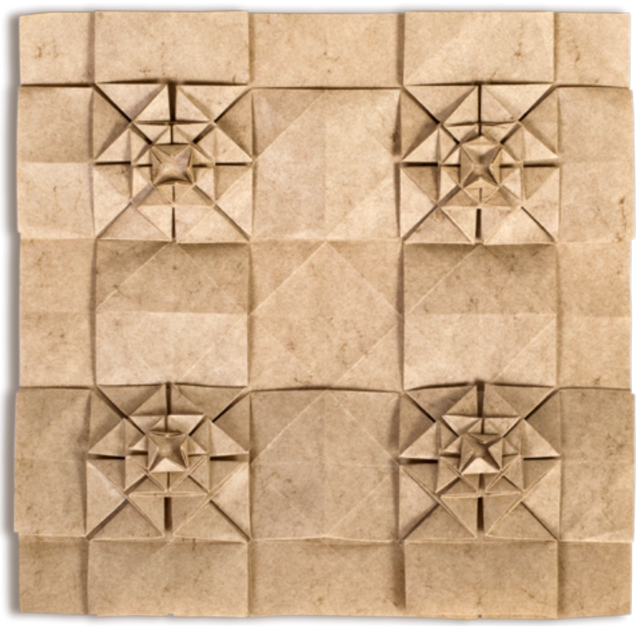


Pineapple tessellation: Recto (top), recto with backlighting (center) and verso (bottom).

Self-similarity tessellations are unique. The idea with self-similarity tessellations is to repeat the same fold, but on a progressively smaller scale. One such model is shown on the next page. The hydrangeas created by Shuzo Fujimoto are classic examples of this type of tessellation.

Notice how the squares alternate their orientation while each layer gets smaller. The center is tiny!

There are also **hybrids** of categories. The most common combination of categories is a tessellation of a self-similarity molecule. In the example below, a repeating pattern is made of four self-similarity models.



Four molecules of the previous model make an interesting pattern.

Why Does It Work?

A tessellation is built from a repetitive pattern. This pattern, the basic building block, is called a molecule, as mentioned earlier.

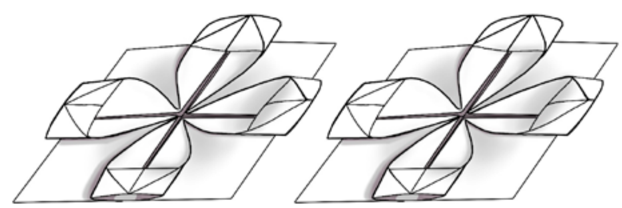
The molecule must have two main properties. This concept is easier to understand if we first think of a tessellation that repeats the same way in all directions.

A. Each molecule must be fully rotationally symmetric, which means it looks the same if rotated by 90° or 180°; it should be a design that has the same pattern of folds on all of its (four) sides.

B. The edge of the single molecule is the edge of the original sheet of paper it was folded from. A design that keeps all the molecule edges on the original edges of the paper also satisfied this rule.

With these two simple rules in mind, it is easy to see that two identical molecules can be positioned, one next to the other, and their edges will match. In other words, if those two molecules are unfolded and spread flat, it would be possible to tape them together and collapse them as one sheet. This is the equivalent of folding the two molecules from a 1:2 rectangle.

Here is an example, using the molecules for my Red Flower tessellation.

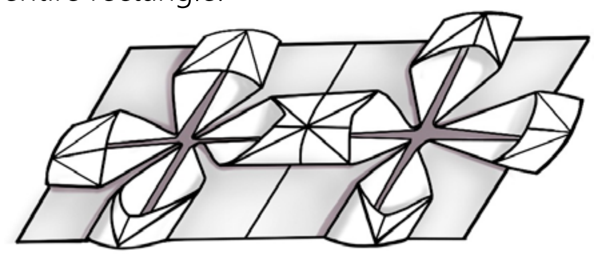


Two molecules, side by side.

If you unfold the common edges (the ones next to each other), tape them together, and then collapse both of them, you get a 1 × 2 molecule tessellation.

Of course, if you can tape together the two, why not simply start with two connected squares—in other words, use a rectangle of 1:2 proportions?

You could execute all folds and collapse the entire rectangle.

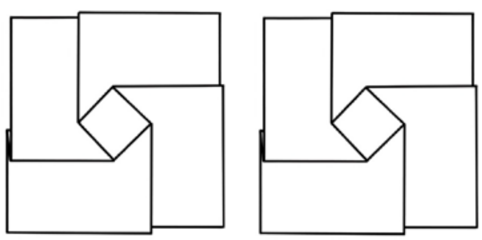


Two molecules form a rectangle.

Mirroring or Shifting

As we just saw above, if a molecule edge is fully symmetrical, you can add the next one just by shifting the molecule to the side.

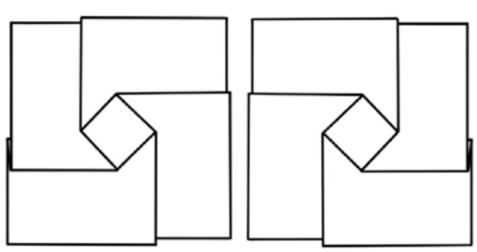
But not all the molecules can be shifted so easily. Look at these basic square-twist molecules:



One molecule has been shifted to the right of the other.

Positioning these molecules side by side won't work, since the mountain folds on the common edge do not match up. The molecules are similar, but the edges do not align.

The folds on the shared edge will align if you mirror the molecule:



Two molecules, mirrored.

Now the mountain folds match up with each other, and it would be possible to fold it from a single rectangle.

These are simple rules, it's true, but they work with just about all classic tessellations.

How to Make a Tessellation: the Three Basic Steps

Step 1—Make a grid.

The grid serves two purposes: it provides reference points for all the precreases and it almost always plays a role in the molecule structure.

Step 2—Add all precreases.

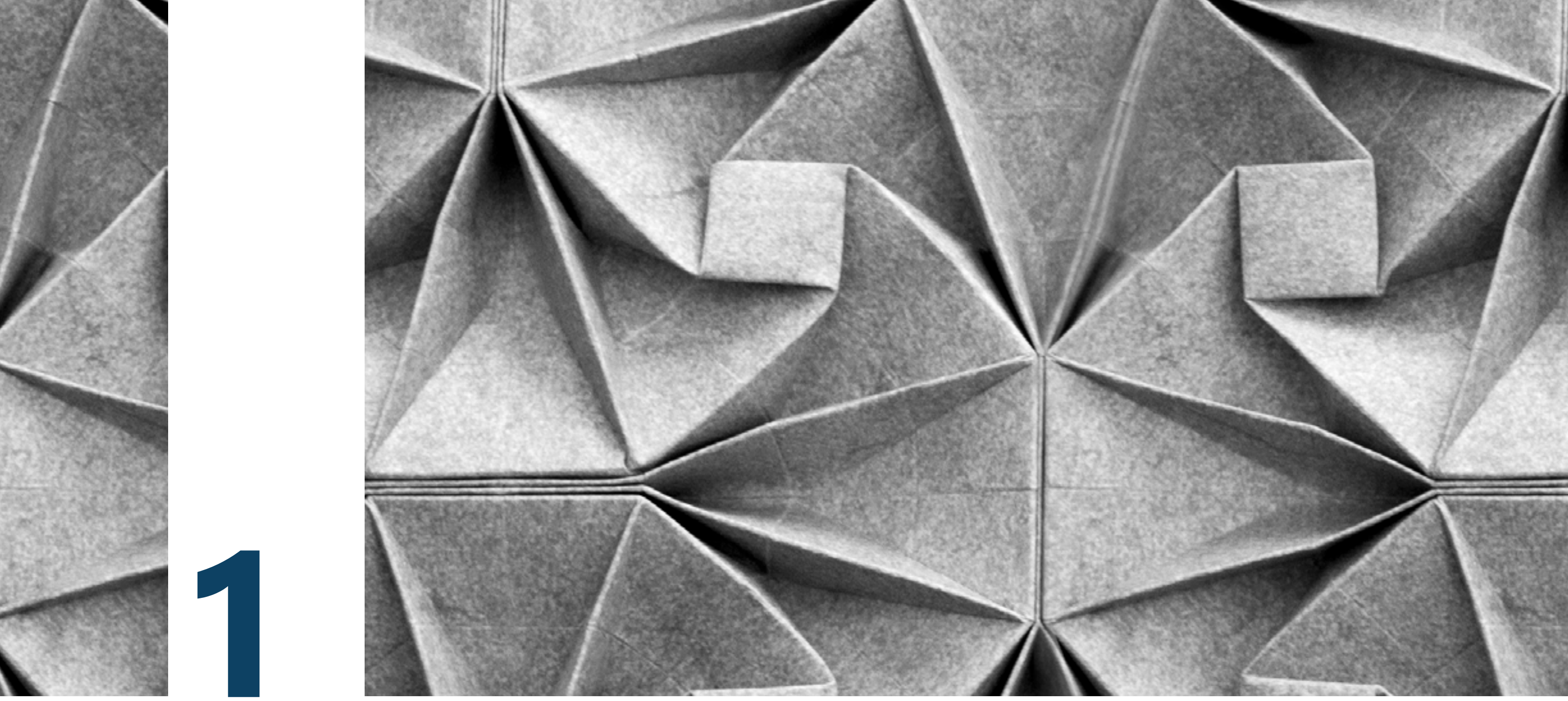
This step includes all the creases beside the grid that are needed to enable a collapse. Most of them are made before you start to collapse. Some, quite rarely, will be added during or after the collapse. Putting in the precreases is usually the longest part.

Some tessellation doesn't need any precreases and are build as we go using the grid lines as references.

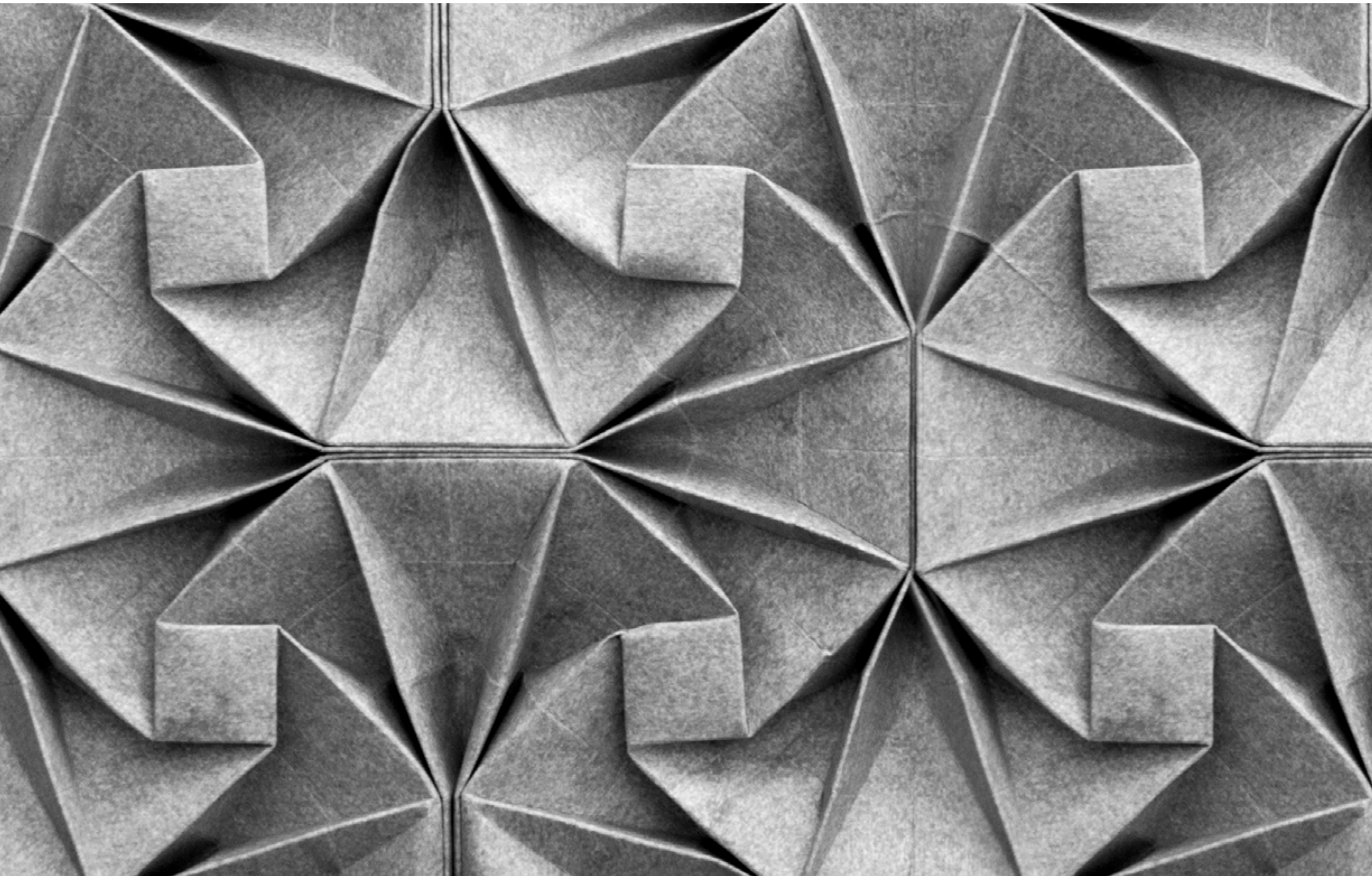
Step 3—Collapse.

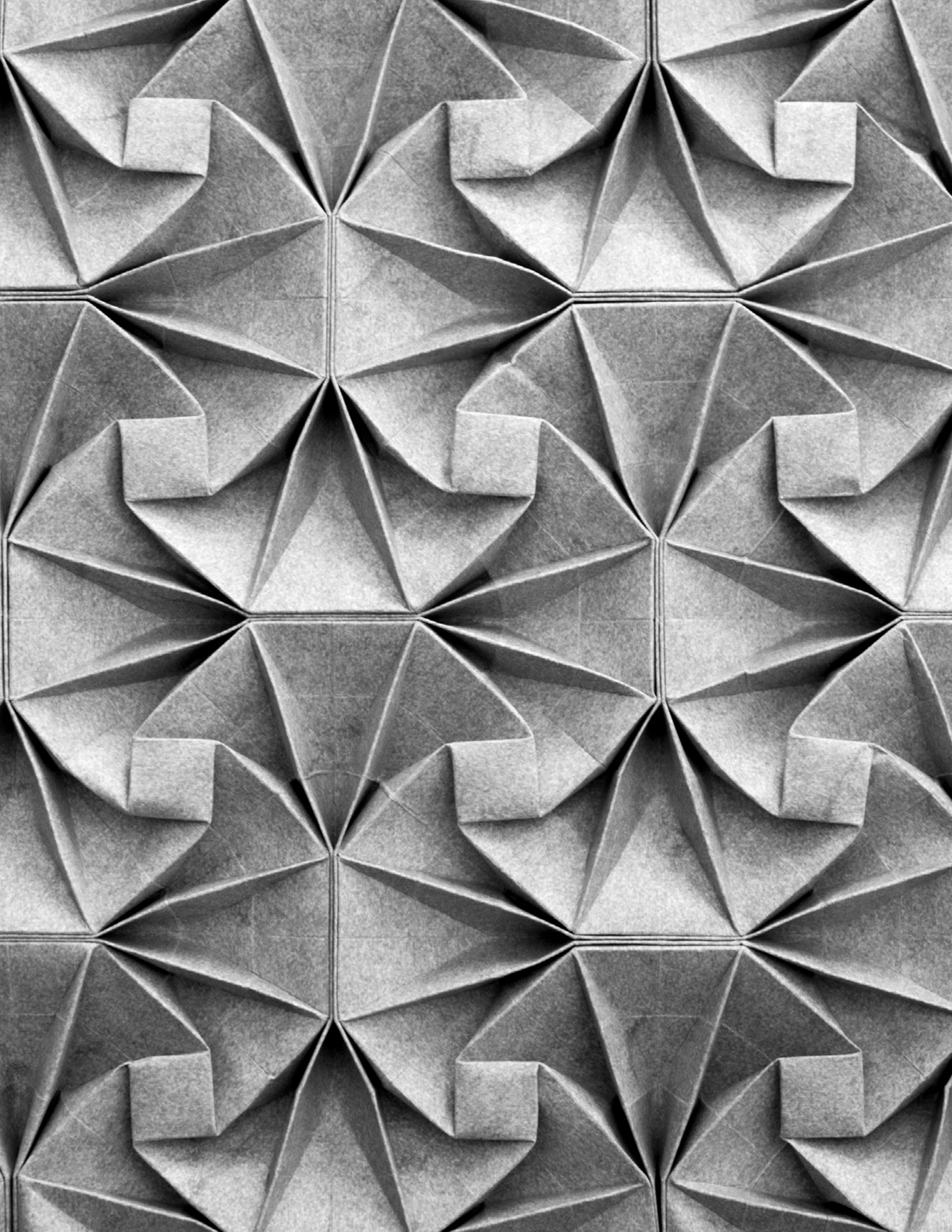
The process of executing the folds to form a three-dimensional state is the most challenging and satisfying step. However, the phase in which part of the paper is still flat while another part is already folded may create a tension that isn't always easy to handle. Just saying.

A single molecule of NSNS (page 138) made from mirror-finish stainless steel.



1 How to Use This Book





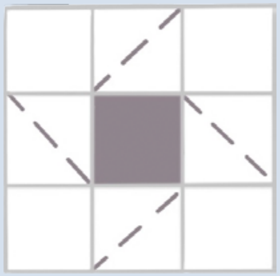
1 1 The Structure of the Book

This book is divided into families of tessellations. Each family is based on a certain principle, which is presented in the first (easiest) model.

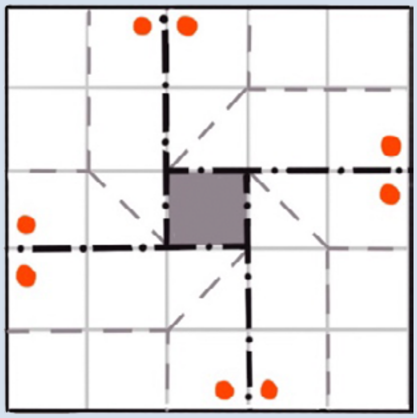
From there I explore some variations on that principle. Every variation is presented in three (or more) stages.

The first stage is always the simplest—*the single molecule*. Learning how to form the single unit will help the folder understand the logic of the molecule and how it collapses.

To keep the diagrams easy to understand, I may not clutter things up by showing all the crease lines that align with the grid. However, they will still be used during the collapse and shown on the complete *crease pattern* (CP) later on.



Single molecule, precreases only and no added edges.

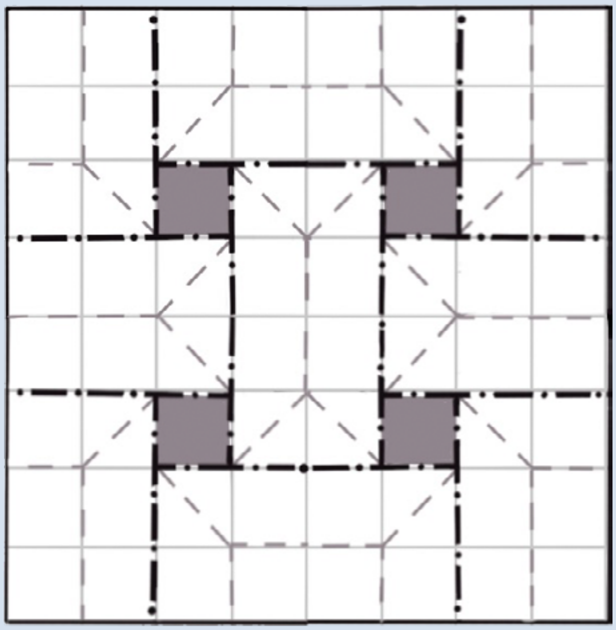


The complete CP (with edges).

While folding a tessellation, it is usually easier to handle it if there's a little extra paper around the edge of the project. A border one square wide goes around the molecule, providing something to hold onto or a place to pinch the sides of the molecules. This extra space is presented on the step-by-step instructions of the single molecule as well as on the larger projects, if needed.

Here I also present how much the molecule shrinks in comparison to the original size of the paper. For example, a shrinkage rate of 5:3 means that the starting size of a molecule was 5 by 5 squares, while the size of the collapsed molecule is 3 by 3 squares.

Next is a *2 by 2–molecule project.* A CP is presented with all precreases needed, as well as a 3D isometric projection of the final project.

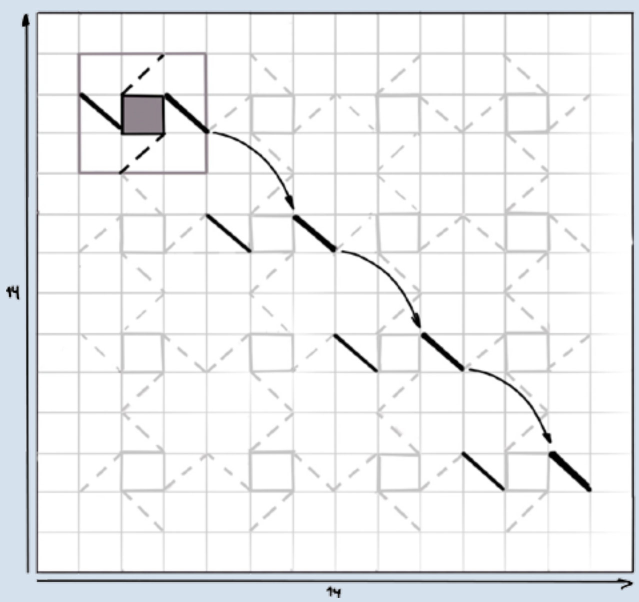


A crease pattern for a 2 by 2 project.

I always include some general tips and advice at this stage, but each model has its own tricks and I present these as well.

At this stage, try not to be sidelined by the beautiful 4 by 4 project that comes next! Practice the 2 by 2 projects and get your skills down first.

Next is the *4 by 4 project*, complete with step-by-step instructions, a full CP, and my recommended shortcuts and special techniques I've developed over the years to make the process as efficient as possible. Images at the beginning of the model's instructions present the 4 by 4 project from both sides, and can help you to understand the mechanics of the collapse.



This crease pattern shows the placement of many molecules and my recommendation for folding shortcuts.

For many tessellations, you do not need to precrease molecule by molecule. There are shortcuts you can take to shorten the workload. Folding molecule by molecule requires too many rotations and movements of the big sheet. It is highly inefficient. Instead, I show you how to align folds from adjacent molecules so they can all be executed quickly, one after the other.

By using these techniques, some of the tessellations may need only a single rotation of the paper! Generally, you fold all the top-left to bottom-right diagonals first, then rotate the paper 90°, and complete all the missing diagonals.

Another important feature of the 4 by 4 CP is the *grid size*. The vertical axis represents the total number of rows, while the horizontal axis is divided into columns with a numerical sequence. For example, the Pineapple tessellation has a grid sequence expressed like this: 1 + 4 x 6 + 1. This tells you that you need a grid 26 squares tall and 26 squares wide, or simply a grid of 26. Proceeding from left to right, there is the extra square, then 4 molecules that are 6 squares wide, followed by another extra square, or a total of 26 squares.

This formula is the way to figure the grid size for any project. A project of 8 by 8 molecules (the molecules measuring 6 squares wide as above) will need a grid of 50 = 1 + 8 x 6 +1 squares.

Here I also present the final size (in squares) of the resulting model.

Last, *Above and Beyond* shows the potential of the molecule, either by showing a large scale model or a special use for the design, like a pendant or wall tiles.

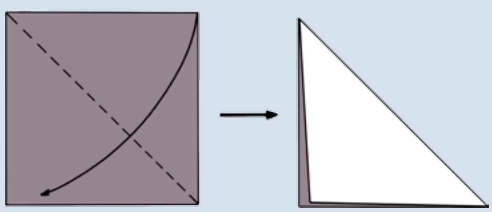
1 2 How to Read the Diagrams

Unlike traditional origami diagrams with arrows and fold-as-you-go instructions, the diagrams here show the slow transformation of a flat sheet of paper into an object with dimensionality. But there are still some symbols that you must learn in order to make the right folds and execute the collapse properly.

Symbols

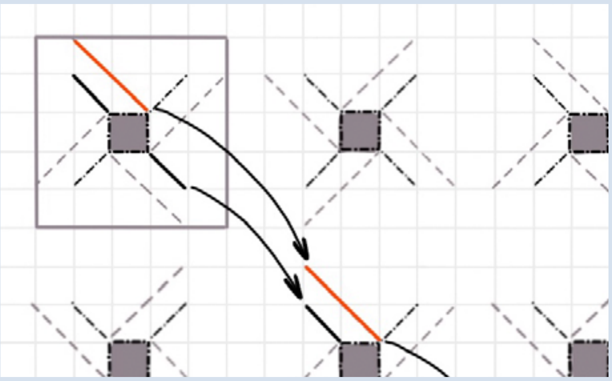
In some models, I use the traditional symbol system:

A *valley fold* is marked with a dashed line. In traditional diagrams, an arrow will accompany it. The paper is always folded forward from the tail of the arrow to its head.



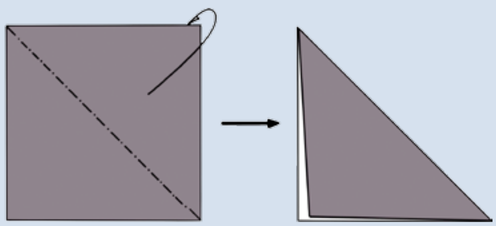
If you are using a sheet that has one side colored and the other white, this process reveals the white side.

I also use *arrows* to indicate the workflow of my recommended shortcuts. In this figure the bold lines indicate where to put the valley folds and the arrows indicate where to skip to next on the same diagonal.

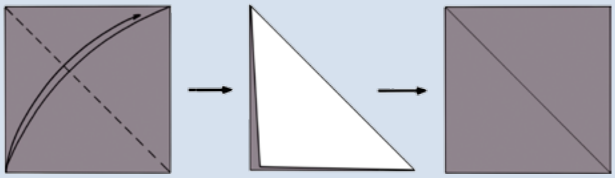


The bottom arrow tells you to skip diagonally across three squares to find the next valley to fold. This is true of all the valleys on the CP.

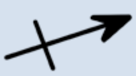
A *mountain fold* is marked with a broken dashed line. In a traditional diagram, a hollow arrow head will accompany it, indicating that you should fold behind.



Fold and unfold is shown using a double arrow, back and forth from the starting point. Note that after the paper is unfolded, a thin crease line appears. The grid creases are always shown with these thin lines.

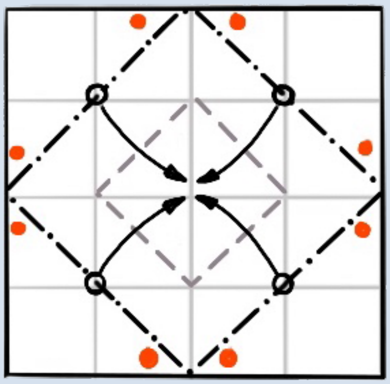


The repeat arrow—an arrow with lines crossing the arrow shaft—means that you should repeat the last command given on the marked area. Repeat as many times as you have lines crossing the arrow shaft.

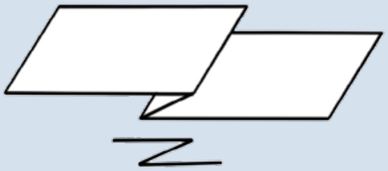


Turn over—a spiraled arrow tells you to flip the paper over to the other side.

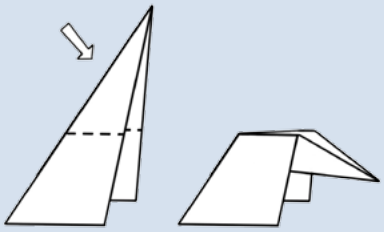
Areas of interest are shown with white circles around them. Red dots are used to show where to *pinch or hold* the paper.



Pleated edges—a zigzag line below the model shows how the edges of the paper are folded. This helps you understand how the edges of the molecule are formed.

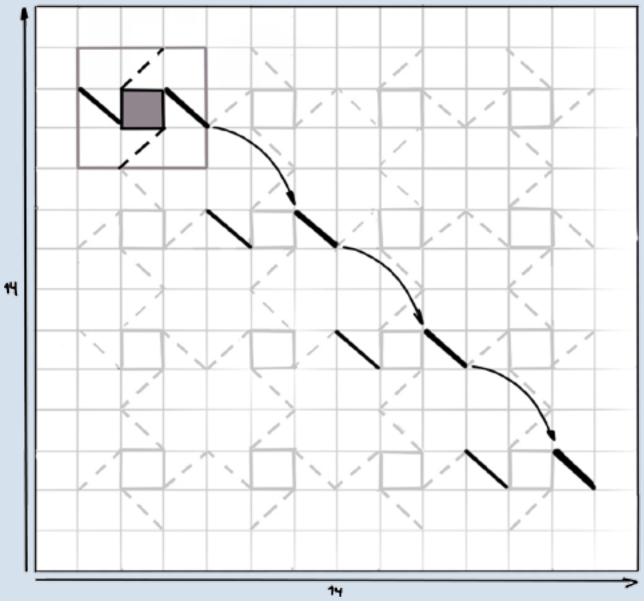


Inside Reverse Fold—a fold line is reversed to switch its orientation. The command is presented by a wide white arrow. Here the upper part of the mountain crease is reversed to be a valley by collapsing inward (hence the name).



1 3 How to Read a Crease Pattern (CP)

A CP holds a lot of information: the size of the grid, the structure of the molecule, the number of the molecules included in the project—all of this can be found in one handy diagram. Valley and mountain fold lines show where to mark the precreases, and the orientation of the folds when you are ready to execute the collapse.



As I mentioned earlier, in many tessellations, there are ways to make the precreasing process much more efficient than molecule by molecule. The idea is to combine creases that are on the same diagonal or straight line in order to minimize the number of times you move or rotate the paper.

These shortcuts are presented in a workflow or pattern that points out which single-square diagonals to fold and which to skip while your hand is moving along the imaginary line that connects them. I can't stress enough that you must fold one square at a time or your accuracy will be compromised. The lines to be folded are bolded and arrows show which ones to skip.

Deciding on the Paper Size

As you will notice, I don't provide a paper size in the diagrams; I leave that for the folder to decide.

All you really need to decide on is the size of the molecule, and from there you can determine the grid size (see later on).

A grid made of two cm (¾") square is a good size for beginners. A one cm (⅜") grid square is more of a challenge. Try your hand at any 2 by 2 project folding molecule of various sizes before you determine which suits you best.

Deciding on the Grid Size

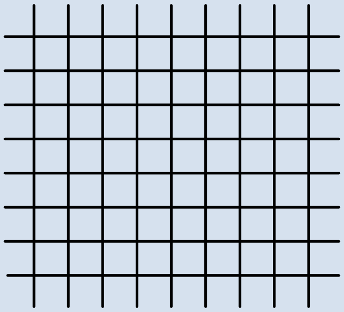
All my 2 by 2 projects CPs show the breakdown of the grid width on the horizontal axis.

Most of the tessellations in this book call for a border around the molecules. This border is usually an extra row and column on all sides. This border has two important uses—it is easier to fold the edge molecule with something to hold onto or clip together, and it brings a nice frame to the finished model.

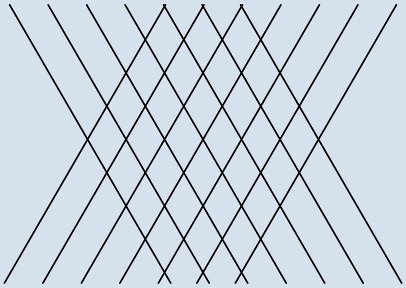
To determine the grid size, you have to calculate the sum of this equation: the width of borders plus (the width of a molecule x the number of molecules). For example, in the figure shown on page 21, there are four molecules and each is three squares wide. Therefore, the total number of squares needed for the molecules alone is 3 x 4, or 12. If we add a row to the left and right, two extra squares need to be counted, for a width of 14 squares. This can be expressed as 1 + 4 x 3 +1 = 14.

1 4 How to Make a Grid

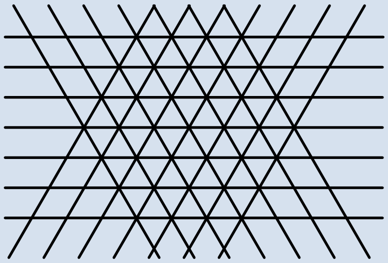
Tessellations are usually made from grids. A grid can be made from squares or from equilateral triangles that form hexagons (which is why these grids are usually referred to as hexagon grids). There are other shapes that can be tessellated to cover a surface, such as a rectangle, or a rhombus, but they are rarely used for origami tessellations. Each unit (square, triangle, etc.) is called a tile.



Square grid

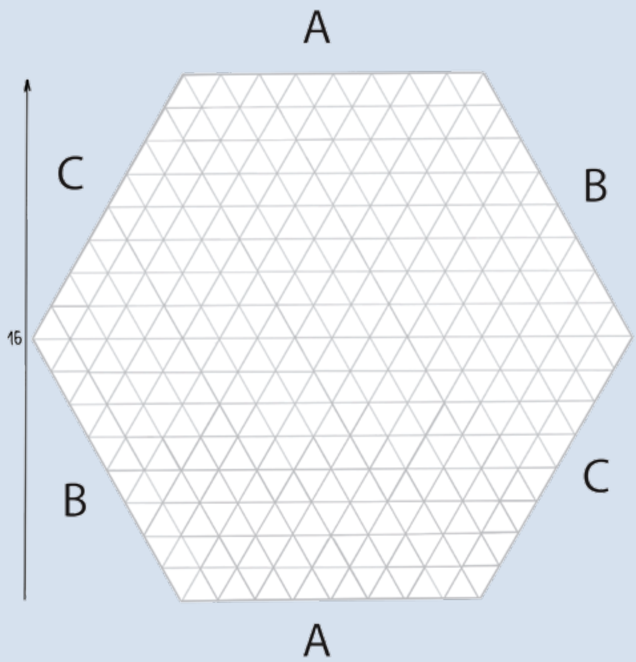


Diamond grid



Triangle/Hexagon grid

Grid size is determined by the number of rows and columns. For a square grid, two numbers are given: 16 by 16, or for short, grid 16. For a hexagon grid, three numbers are given, one for each pair of parallel edges.

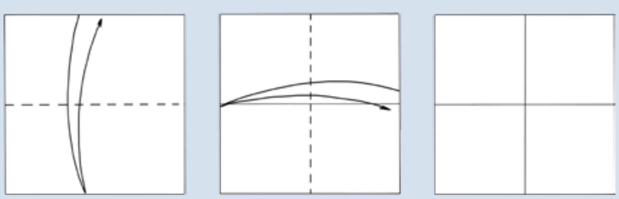


This CP shows 16 rows (15 divisions) between every pair of edges; AA, BB and CC, so the grid size would be 16 x 16 x 16.

Folding a Grid

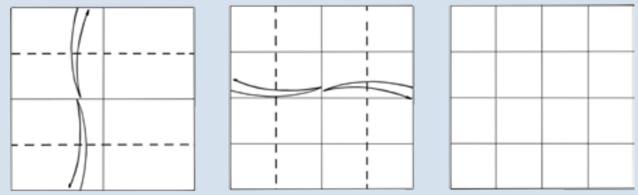
Making a grid may seem like a fairly simple undertaking, but the truth is, it requires quite a lot of skill and mastery.

To get a 2 by 2 grid, you need one horizontal crease and one perpendicular to it. That's easy enough to do—just divide a square sheet by folding it in half in both directions.

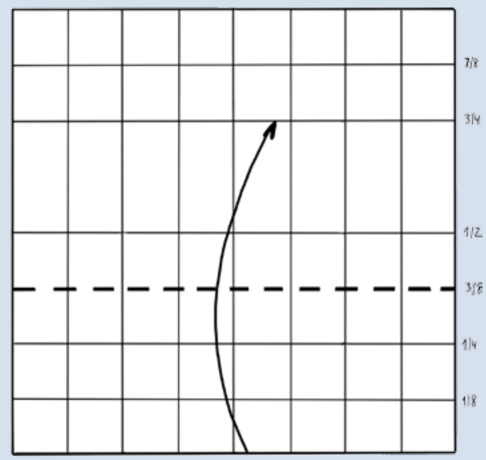


The process of folding 2 by 2 grid.

To get a 4 by 4 grid, divide all halves in half again. This is still easy; this is also known as a cupboard fold (done twice—horizontally and vertically).

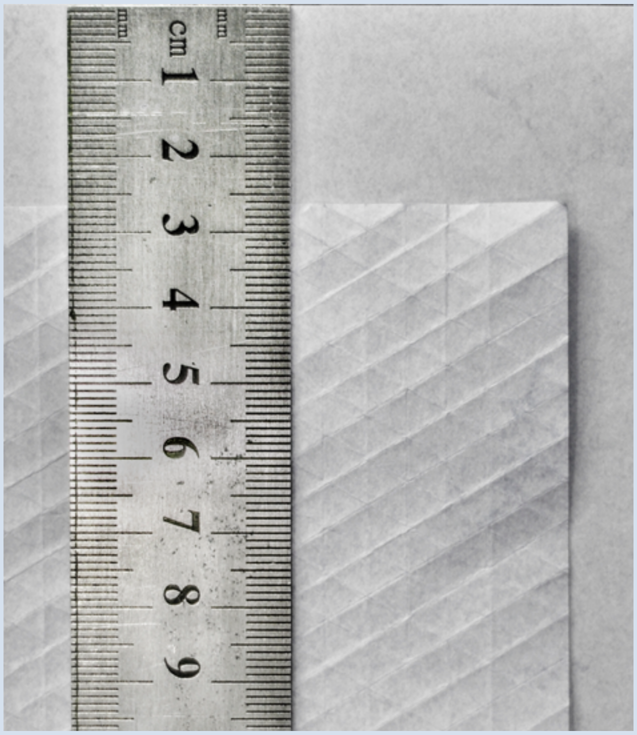


Now try an 8 by 8 grid. Getting the first 1/8 line is easy; you just fold the edge to the 1/4 line. But how do you get the 3/8 line?



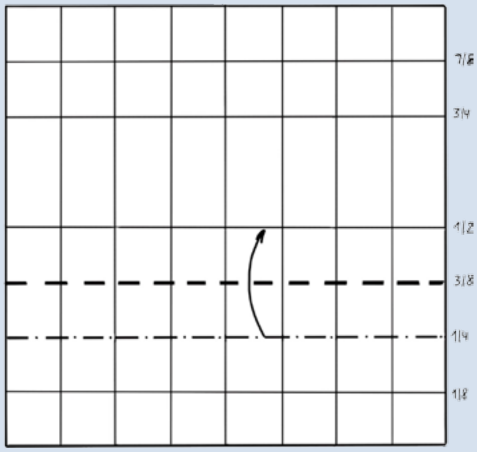
One solution: fold the edge to the 3/4 line, which divides this length to 3/8. This works well enough for an 8 by 8 grid, but it gets worse the more tiles a grid has. That's because every crease shrinks the paper a bit—you lose a tiny bit of paper to the crease. For instance, when you fold the edge to the 46/64 line to get the 23/64 line, the distance of the 46th line is not 46/64 of the paper, but teeny bit shorter than that.

The photo on the next page shows how much of a difference this can make when you have a lot of creases. A full sheet of paper after folding a 128 by 128 grid is clearly shorter and narrower than the unfolded paper beneath it, and yet both sheets are aligned with each other at the bottom left corner.



The corner of a grid 128 sheet on top of an unfolded sheet.

It may seem negligible, shrinkage of just 2.6%, but it does have a major impact. Suddenly, squares are no longer squares, and diagonal creases do not bisect the corners anymore.



The diagram here shows a better way. In order to get the 3/8 crease, pinch the 1/4 line and fold the edge to the 1/2 line. To get the 23/64 line, pinch the 22nd line to the nearest crease.

There are different methods to fold a grid, but this is my favorite!

How to get odd grid sizes

You may need a grid of 14 by 14, 46 by 46, etc. Since the presented method is good for 16, 32 and 64 divisions, we need to find a way to make grids that are not based on multiples of two.

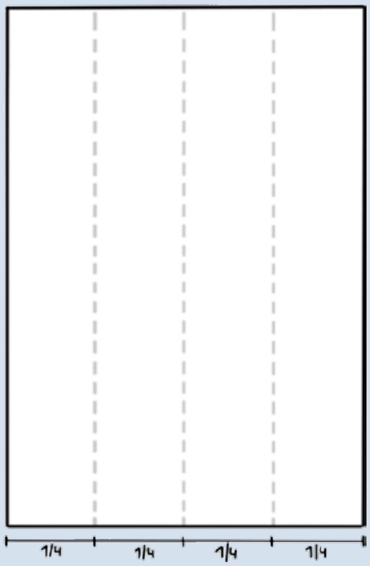
If your grid size is a little under the power of two numbers—for example, a grid of 30—make a grid of 32 and cut off the extra squares. Or you can just have a wider border around your final model.

For the rest, use a ruler to mark the nearest power of two number. If you need a grid of 34, and you use 1 cm for the square size, mark 2 cm from the edge, which divides the paper into 2 squares plus 32 squares. Fold the 32 divisions first, and add the last one to divide the 2 cm gap when finished.

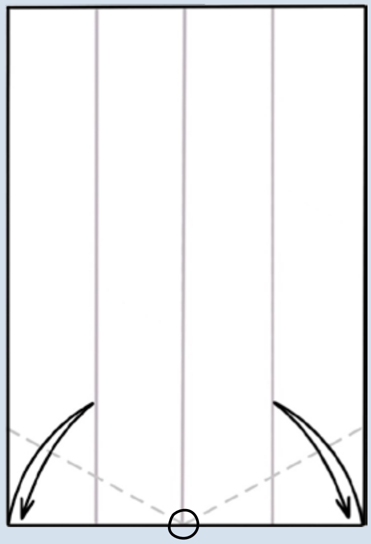
Tips

- I always make my creases bidirectional. It is double the effort, but it is often very helpful in the collapse phase. At the end of folding the grid, make sure all your creases have the same orientation (mountains or valleys).
- Rotate the paper after every division. Since it is quite difficult to make a perpendicular crease against 64 mountain fold lines, do not crease 64 horizontal creases and then rotate the paper to complete the perpendicular ones. Instead, divide a side in half and rotate. Divide the new side into quarters and rotate. Back to the first side, divide it into 8ths and rotate. Divide into 16ths and rotate, and so on.
- The same logic applies to the hexagon grid.

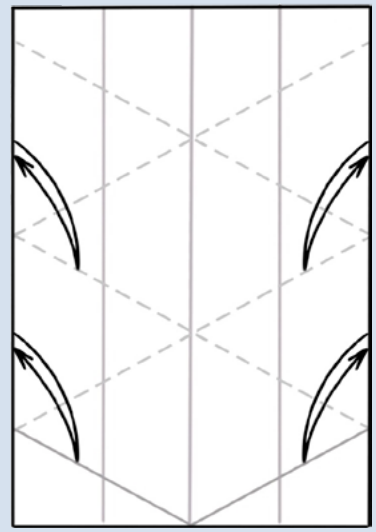
15 How to Fold a Hexagon and Hexagon Grid



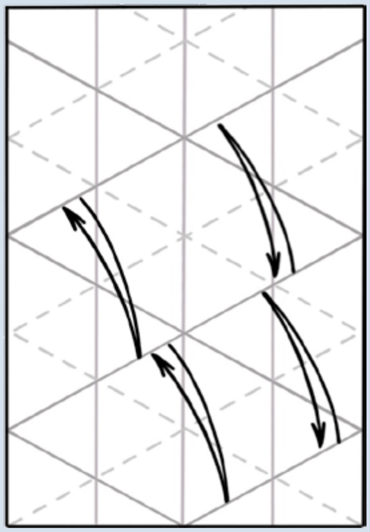
Start with a sheet of printer paper in portrait orientation. Letter size or A4 will both work fine. Divide the sheet into four columns.



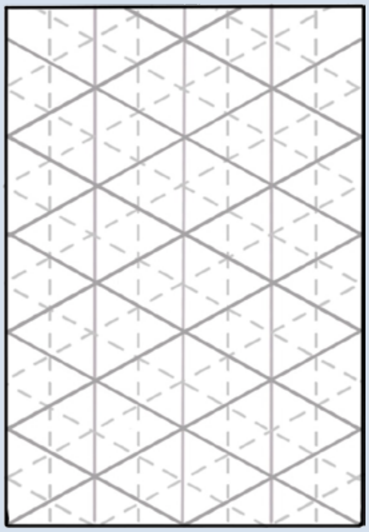
Fold and unfold the bottom corners to their nearest fold lines. Make sure the new crease starts where the center fold line meets the bottom edge.



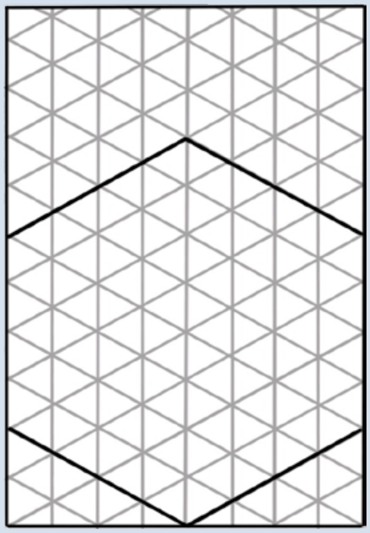
Fold the edges of the paper to the creases you just made. Now you can see the hexagon forming. Fold the edges to the new creases you just created as well. This completes the top edges of the hexagon.



Add creases between every two fold lines to make more tiles.



Repeat.



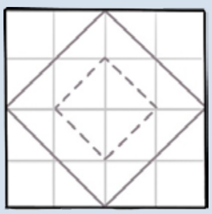
The marked hexagon is divided into a grid of 8 x 8 x 8 (meaning there are 8 rows between every pair of edges).

1 6 How to Precrease

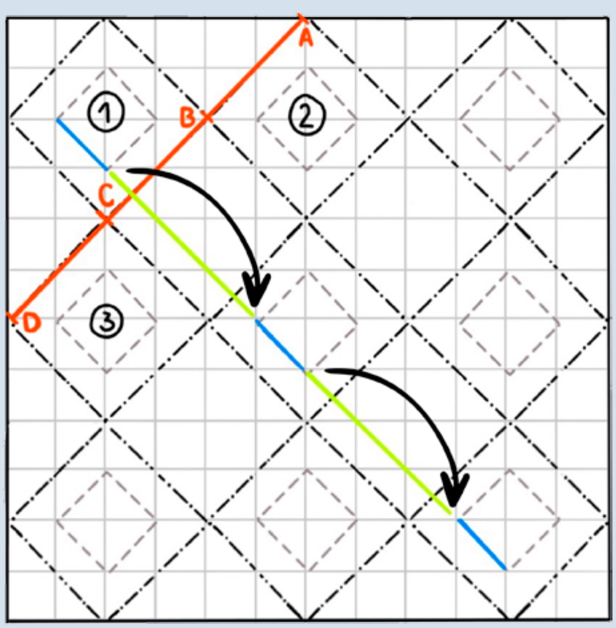
Unlike traditional origami, tessellations are mostly based on precreases that you complete before you start to fold. Some have a step-by-step way to get there, but for most, you will have to prepare all the folds in advance and then collapse the model.

For most tessellations, this is the most time-consuming phase. The most basic molecule in this book needs four creases; usually there are many more.

Luckily, there are shortcuts for this process. Many creases will align and can be folded in one stroke.



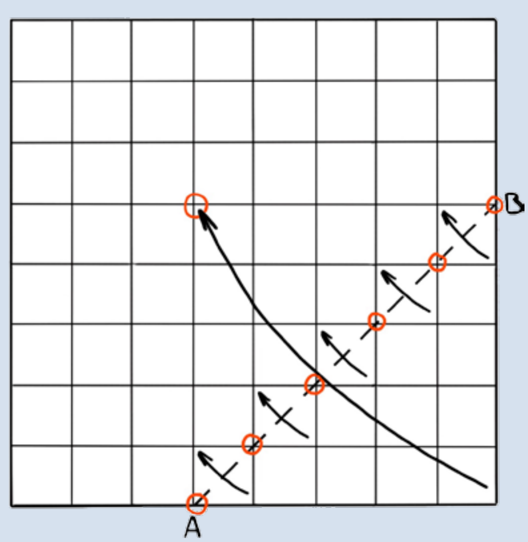
To the right is the single molecule of the Red Flower tessellation. Below is the CP of a 3 by 3–molecule project. Notice that molecules 1, 2 and 3 in the CP below all contribute to one straight line, AD (shown in red): the top-left fold line of molecule 2 (AB); the bottom-right line of molecule 1 (BC); and again the top-left fold line of molecule 3 (CD). So folding line AD in a row gives you a major shortcut.



Finding a shortcut is a bit harder for the inner squares that do not form a connected diagonal. Here we move to another method: counting. All three blue lines are aligned, so if we start with the blue line of molecule 1, we count three squares (marked in green) to skip to get to the next square with a diagonal to fold (shown in blue again). So this is the pattern of work: crease one (diagonal), count/skip three (diagonals), and crease one, etc. The black arrows indicate the skipped diagonals.

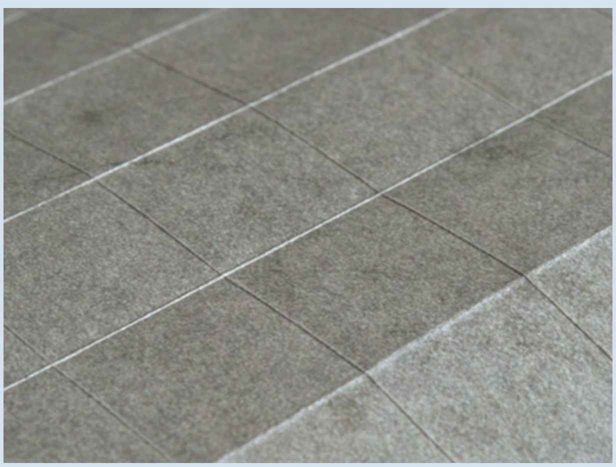
IMPORTANT: Long diagonal creases need to be folded one square at a time. I can't stress this enough. That means you should crease from corner to corner across one square before moving on to the next part of the diagonal. This is tedious, it is true, but the increased accuracy is worth it!

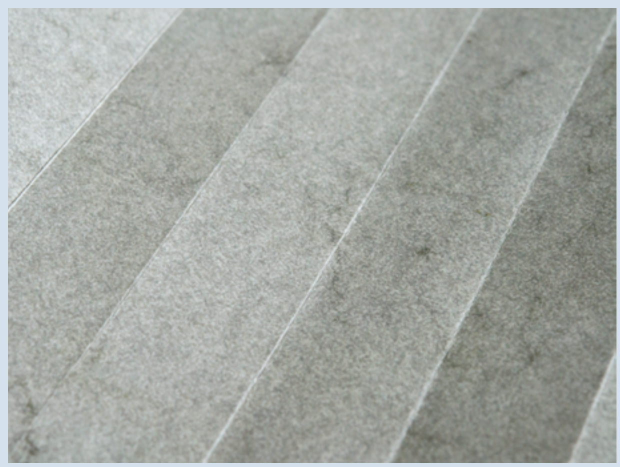
For example, to fold line AB you can locate the lower right corner at the grid-cross (in the big red circle) as a general guide only, but when you mark the fold, refer to each single square as one—fold from the corner of it to the opposite one, and move to the next square (in the small red circles).



Tips

- Make only mountain folds (valley them second to make them bidirectional). If you are asked to make a valley, turn the paper over and fold a mountain. Why? For short point-to-point creases, mountains are much easier—and more accurate—to fold than valleys.
- Once you start working on larger projects, you'll find that you will occasionally make mistakes like miscounting, jumping over the wrong fold line, etc. Don't feel bad. It's pretty much a cosmic law that you will experience this. To avoid the problem, mark the center of each of the molecules with a pencil or bit of Blu Tack (or some other reusable adhesive putty—you can find different varieties at hardware or office supply stores).
- Remember not to make a precrease in the first and last row or column! If you are folding this area, you misstepped somewhere.
- If you follow my advice and make all the grid lines bidirectional (both valley and mountain), you will see there is one side of the paper where the creases are more evident. You can use this side to mark your precreases where it is easier to see the grid.





Ridges left by folding are evident on the left ("top" side), while they are hardly seen on the right ("bottom" side).

1 7 How to Collapse

"*I can't do it! I just can't do it! Oh, maybe I can.* (Deep breath) *Oh, yes... I can!*" I can hear you now.

Yep, the collapse is the stage that really tests your sanity. You hear the phrase "It can't be done!" over and over in your mind again, while the cat jumps whenever a cry of frustration comes out of your mouth. But trust me—it can be done. Look at the final image of the model in the book to believe! If I can do it, so can you!

Keep in mind that collapsing a model made from a square grid is different from a hexagon grid.

With a square grid, you start at the upper left corner and complete one molecule. Then add another one to the right, working one molecule at a time to complete a row.

The first row is simple. The first molecule on the second row starts easily enough, but the second molecule is surrounded on two sides with folded molecules and the other two sides are flat paper. This is where the difficulties lie. If you can manage to fold that second molecule, you can do the rest!

With some tessellations, it is good to partially collapse all molecules in a row before collapsing each completely, one by one. This saves a bit of work by unfolding creases that will be in your way for the next row.

For a hexagon tessellation, I recommend that you start from the center, making one molecule that the others are clustered around.

Then you make a ring around the center with six molecules. The last one (and sometimes even the fifth one) may give you some trouble, as it coincides with the first one and involves unfolding some pleats made earlier to allow you to work on the paper. When opening the paper, you need to be careful not to unfold the adjoining molecules too much, but some unfolding will be necessary to complete that final molecule. If this proves too difficult, try to fold the fifth and sixth together instead! The folding continues in this fashion, building rings around the center.

Tips

- Take heart! Do not give up! The folder is stronger than the paper.
- Use clips (see the next section) to hold folds and edges in place, if needed.
- There is literally always another side with an origami tessellation. Many times it is easier to work the collapse from both sides of the model. The structure might be more visible to you from the other side and so even easier to handle. Give it a try.
- When folding a tessellation, sometimes you work out that there is a special rhythm or pattern of work that makes things easier. Write it down the moment you realize that sequence. It will help you tremendously the next time you try to fold it. For example, for the Cubes tessellation, while finishing a row during the collapse, there are mountain folds that go all the way to the bottom of the sheet. The pattern is that two mountains are joined together, and the next pair is two squares away. When you collapse, look for those pairs.

18 The Right Tools for the Job

Clips

Metal and wooden clips.

Clips, either metal or wood, are a must! The more, the better. The wood ones keep your paper unmarked, but they are usually not as strong as the metal ones. Just be careful, because some of the metal ones are too strong and leave their marks on the paper. Metal binder clips can be found at an office supply store. Tiny clothespins can be found at craft stores in the scrapbooking section.

Bone Folder

Various types of bone folders.

A bone folder (probably really made from bone at one time) can be used to press the paper into nice, sharp creases. There are many types—wood, metal or plastic—and they help to ease the fatigue in your hands from folding a lot. My favorite is the middle one, in the photo below left, bought from Origami-Shop (www.origami-shop.com). It is made of a very sturdy wood, easy to hold, and the pointed tip is very helpful in tight spots. Various types of bone folders can be found at craft and/or art supply stores.

Reusable Adhesive Putty

Blu Tack can be used to mark tiles.

I use this to help me keep track of where I am during the long precreasing process. I like to use tiny bits to mark the centers of molecules or help me block off the border rows and columns. You could always use light pencil marks, but the putty is easier to see. It can easily be removed without any residue, and will help you stay oriented throughout.

Scoring Tools

Scoring is a procedure that weakens the paper where you want a fold to be. You use tools to depress lines in the paper and this provides a folding guide for later. You can buy a scoring tool at a craft store, but it is just as easy to use a pen for that (just be sure that the cartridge is empty of ink first!). Use a ruler or some other kind of straight edge to score against to help you keep your work straight. You can also use a machine called a plotter to help you score very complicated patterns. The masters even use laser plotters! For our purposes, though, a hand tool will be all you need. This book contains only one tessellation that requires hand scoring (Rounded Cubes, page 51).

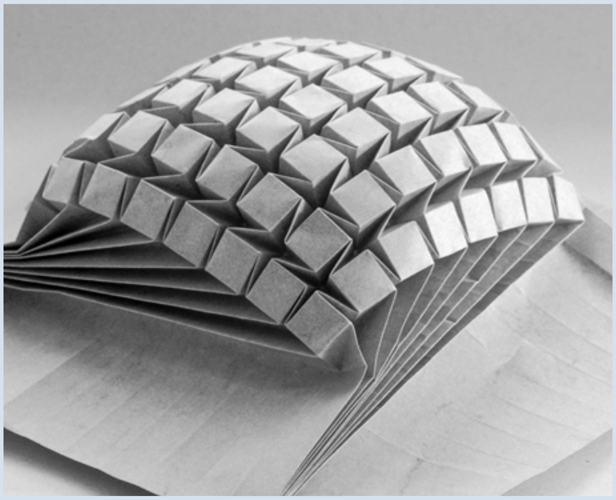
Cotton Swabs

Cotton swabs are helpful to puff and stretch paper molecules that include a pocket, or a rounded face, without tearing it. Any swab with a soft tip works well.

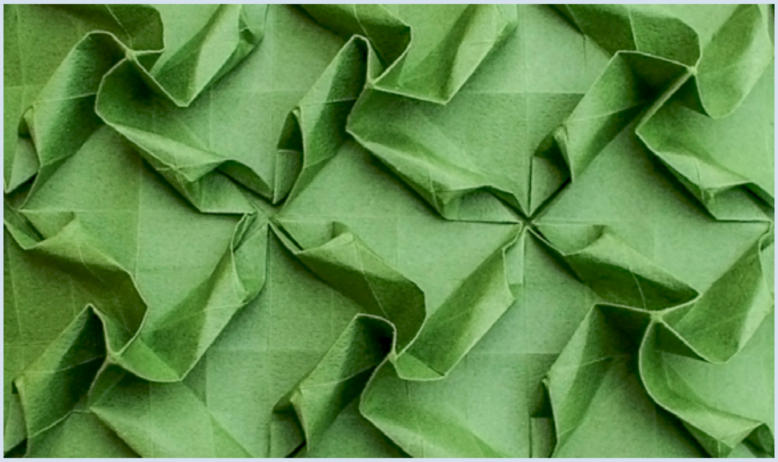
1.9 Papers I Love

Elephant hide is probably the best paper for tessellations. It has a great "memory": creases stay sharp and the paper tends to keep its new location after the crease was made (in other words, it doesn't come unfolded so easily). It also reverses well.

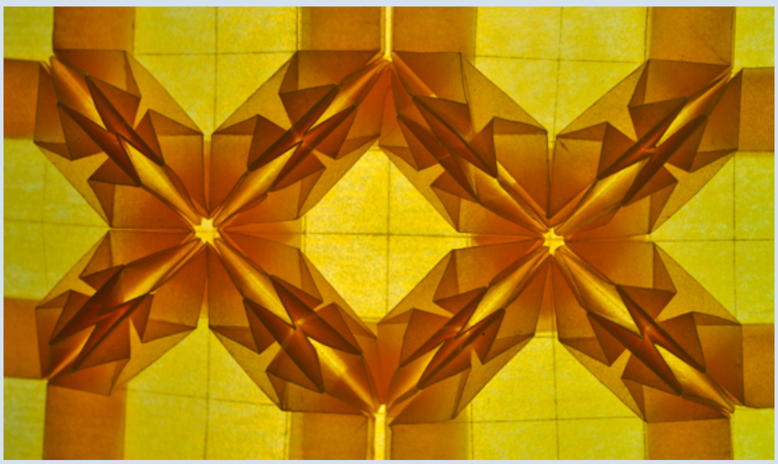
Its weight is 110 grams per square meter (GSM), which is the standard way to measure the weight of a paper type (you will even find it on the package next time you buy a ream of printer paper). Normal printer paper is around 90 GSM, while simple kami—the most popular kind of paper used for traditional origami, is around 50 GSM.



But back to elephant hide: the thickness is 0.1 mm, which makes it highly durable and hard to tear. Nowadays there are only seven colors in a pale palette. The light shades are translucent enough to allow the light through and show the beauty of the layers.



Tant is thinner paper (80 GSM), but still solid enough to hold the creases well. Its best property is that it comes in 100 colors. It is not as translucent as elephant hide.



Glassine is the thinnest paper I like to use at 40 GSM. It is best for small projects. It is easy to tear and very hard to reverse. The memory, though, is almost perfect. Its best feature that it allows light through beautifully and creates a great play of light and shadow.

1 10 Understanding the Way Variations Are Made

Once I get a molecule down, I like to try to find variations. The Cubes molecule (see page 36) is a great example. I created a whole series of interesting models just by looking for the variations.

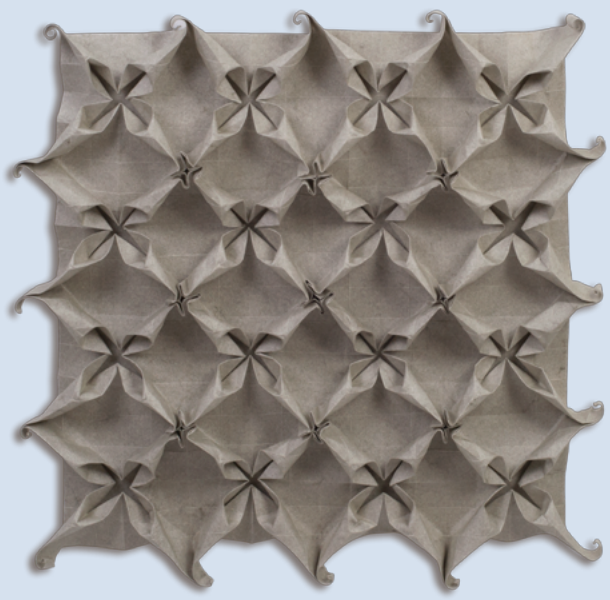
Finding a variation can be done in two ways: you can play with the folds and physically experiment, or you can explore the possibilities directly on the CP.

First, try changing the order of the paper layers. Many tessellations are constructed of paper layers and when you change the order of those layers, you will find new models. Change the direction of the fold, or switch from a square grid to a hexagon grid, but use the same overall folding procedure. Or try repeating procedures on a smaller scale.

Red Flower tessellation.

Double Red Flower tessellation repeats the procedure on a smaller scale.

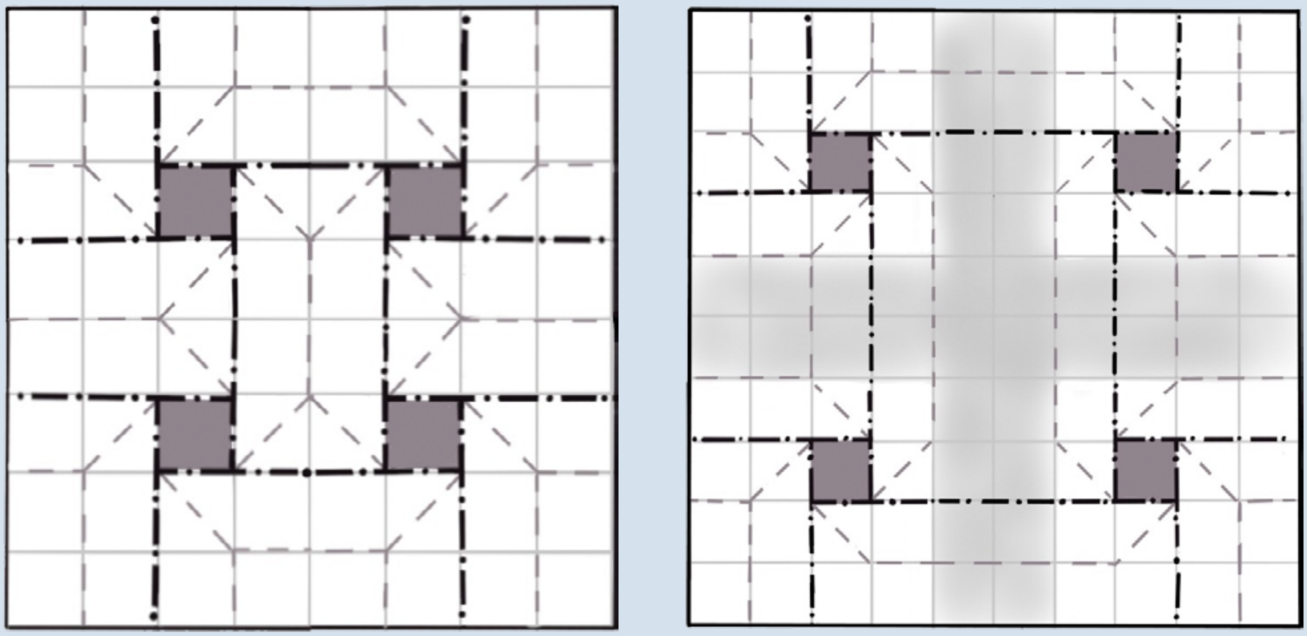
Hexa Flower tessellation uses the same procedure on a hexagon grid.



Green Flower tessellation is similar to the Red Flower tessellation, but the molecules are rotated 45°.

To use a diagram as the basis for variations, start by studying any tessellation CP. A simple way to do this is to print out a CP and actually cut out the molecules. As long as you maintain the rules of the molecules, you are bound to make tessellations out of your efforts. Take a look at the Cubes CP on the next page; notice how the molecules are connected (two crease lines make a V shape). Now see what happens if you cut out one molecule and paste it one or two rows to the left, adding a space between the molecules. I call this process *spacing*.

Using a two-row spacing is exactly how I developed the Wave tessellation from Cubes.



Comparison between Cubes (left) and Waves (right).

Another simple method is to change the length of the crease lines. What will happen if a diagonal crease crosses two squares instead of one? The result is very different, and it is actually hard to fold. But if you add a one row and one column around each new molecule—voilá, you get Bricks!

You can also try playing with angles. In Cubes, all diagonal folds are 45°, going from corner to corner of a square. But if the diagonal is of a rectangle, at a ratio of 1:2, the angles of the folds change and it results in a lovely corrugation. I named it Quadilic.

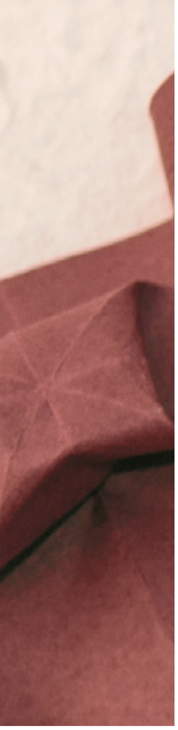
Another variation is altering the line's nature. Instead making them straight, they can be curved. Rounded Cubes is a great example of this.

You can change other parameters as well: the size of the inner square and its shape (rectangle instead of a square). What others can you think of?

1 11 The Process of Original Creation

The process of creating has always been an indescribable experience for me. This is the part that cannot be boiled down into lines of code. But understanding the logic and rules of tessellating will help you develop your own process. And that comes with practice.

So here is my advice: Start simple. Dare to play around. Dare to doodle with paper; grid it, mold it, and then try again. And again. Don't be afraid to make mistakes. Ask yourself what will happen if you try something new or different. Once you allow yourself the freedom to experiment, the creative process will get easier, and your own designs will most definitely follow!




3

The Red Flower Family

Introduction

The Red Flower family is based on a 4 by 4 grid with a tilted square that connects all the midpoints of the edges. The one step to join all those points in the center of the grid is the essence of the process, and this chapter revolves around it. The name came from the model's flower-like look and the paper I used while discovering it—a red tant.

Variations are based on duplicating the process to get an inner flower, rotating the CP on the grid, and using the same steps on a hexagon grid.

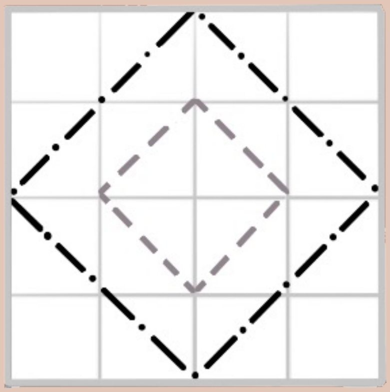
I am not the first to find this model, but I believe the variations are unprecedented.

3 1 Red Flower

Right: recto view of a 4 by 4–molecule Red Flower tessellation.

Verso view is below.

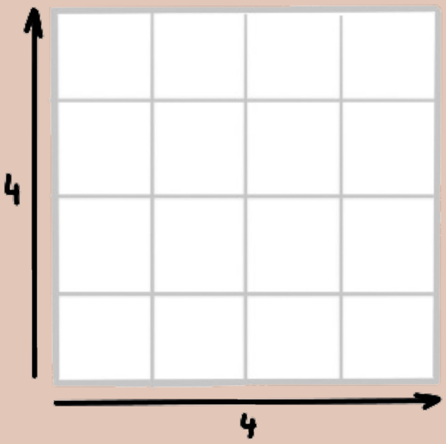
The Single Molecule



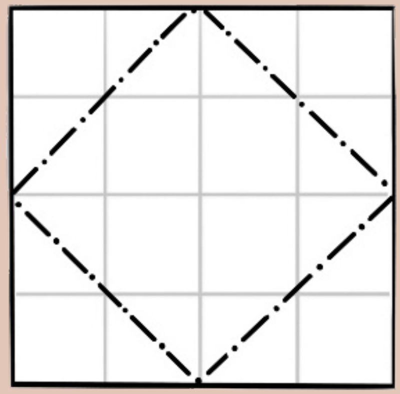
On a 4 by 4 grid, eight crease lines create a square within a square. Since every molecule is symmetrical, you only need to shift the molecule to tessellate it.

Be warned: this model blooms only on the last step!

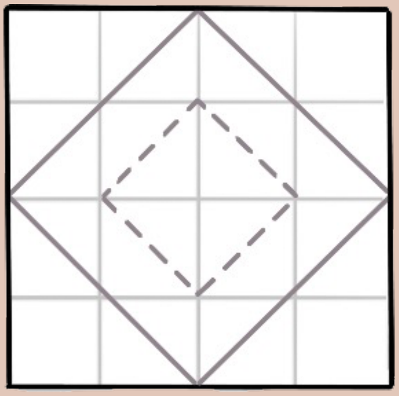
The shrinkage ratio is 2:1.



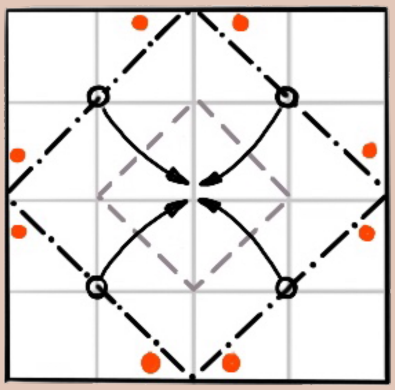
For this model, there is no need for an extra row or column around the molecules.



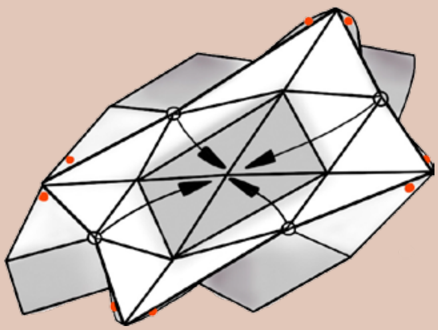
Mountain fold a tilted square connecting all the midpoints.



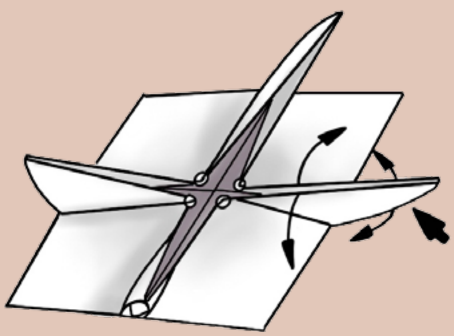
Add a smaller inner square with valleys. It is easier to fold these by pinching the mountain lines and aligning them with the center point to mark the valley folds.



Collapse by pinching all the corners of the tilted square (marked with red dots), while gathering all the midpoints at the grid center.

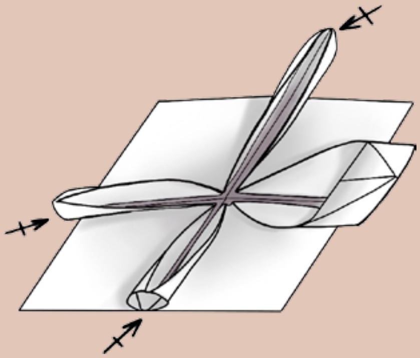


In process.

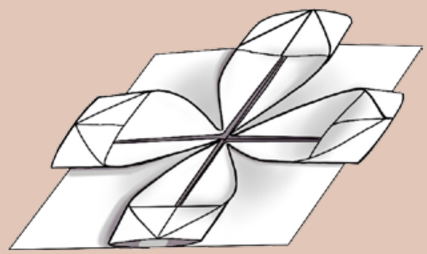


Fully collapsed.

Shape a petal by spreading the "lips" of the standing triangle.

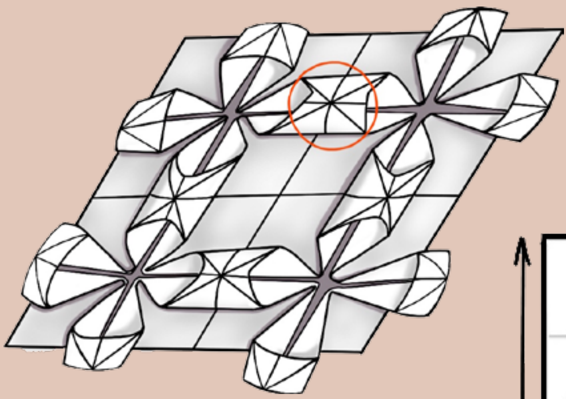


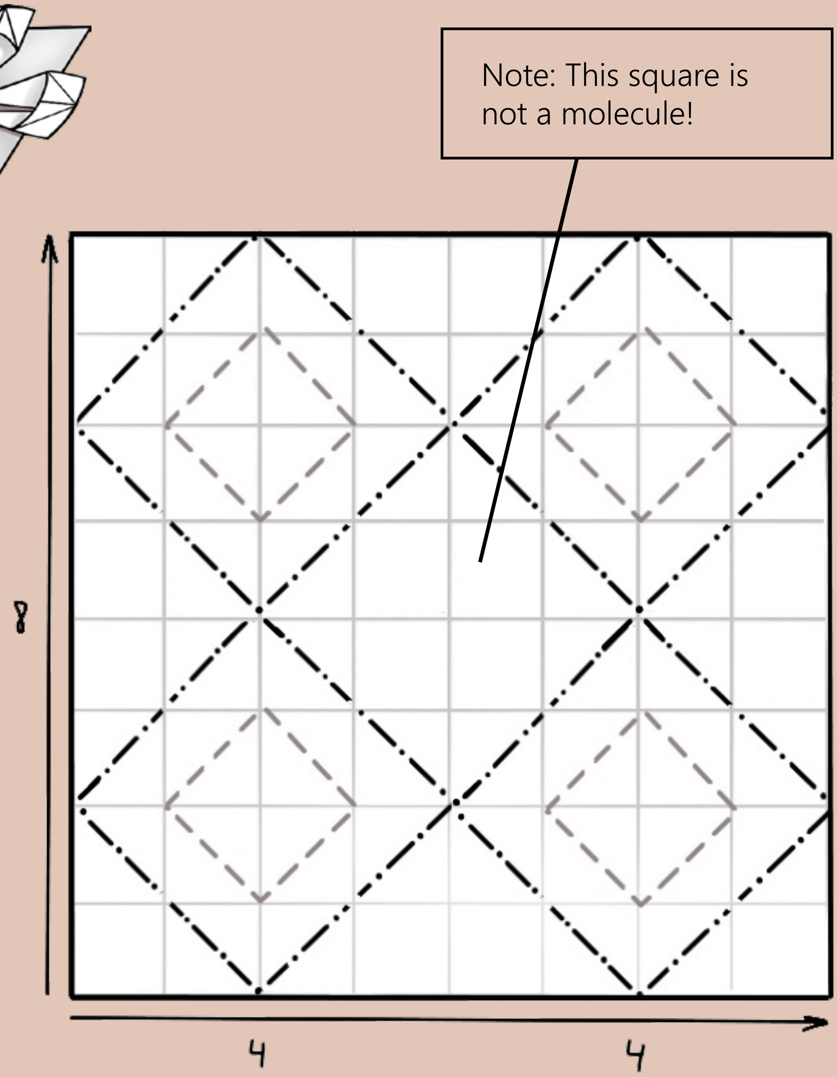
Repeat all over.



The finished molecule.

2 by 2 Molecules



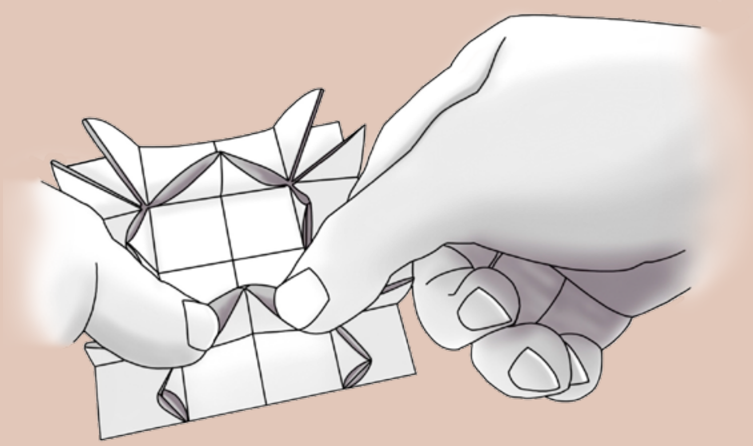


Use a grid of 8 by 8. Add all precreases.

Note that the mountain fold lines of the outer squares align with each other and can be creased with one stroke.

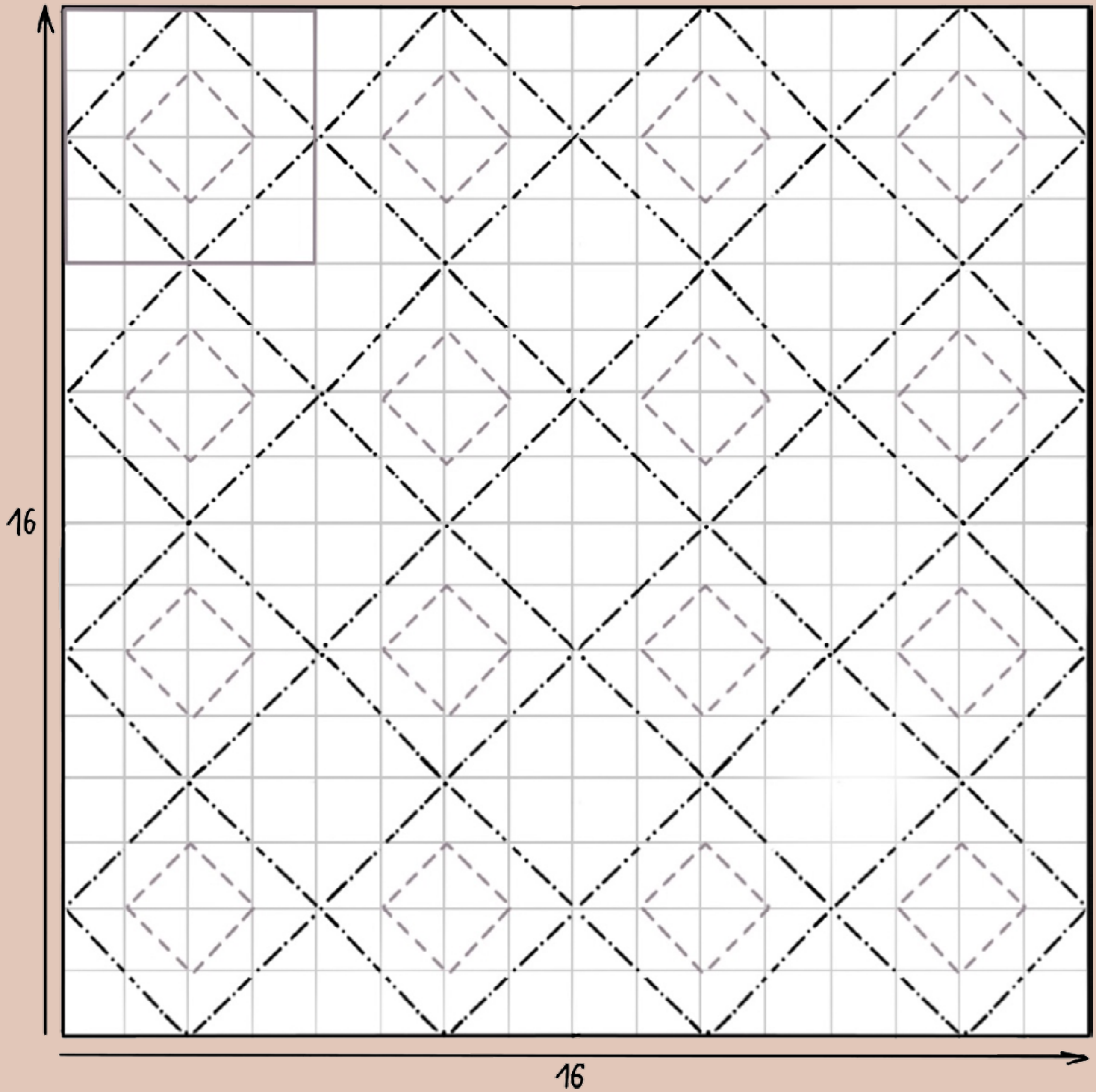
Be sure to keep the center square free of folds. It is not a molecule.

Collapse the top left molecule first and then add the right one. Try to collapse the bottom two together.



Unlike the single molecule, this project has petals that are not on the edge of the paper (marked in red, top left drawing). To puff them, use both thumbs, inserting them into the pockets and spreading out the lips, causing the point to spread to the sides.

4 by 4 Molecules



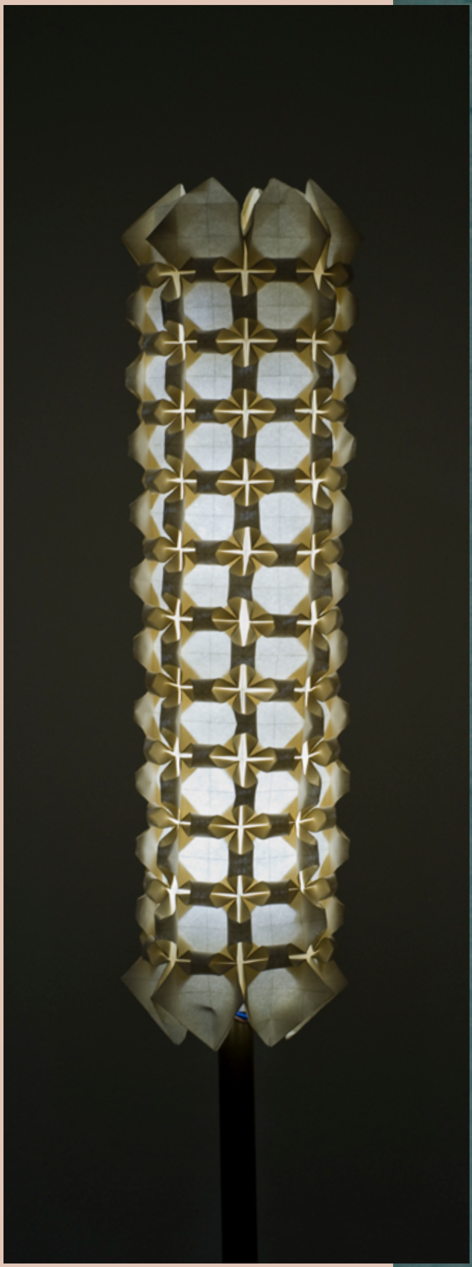
For a 4 by 4–molecule project, start with a 16 by 16 grid: 4 x 4 = 16.

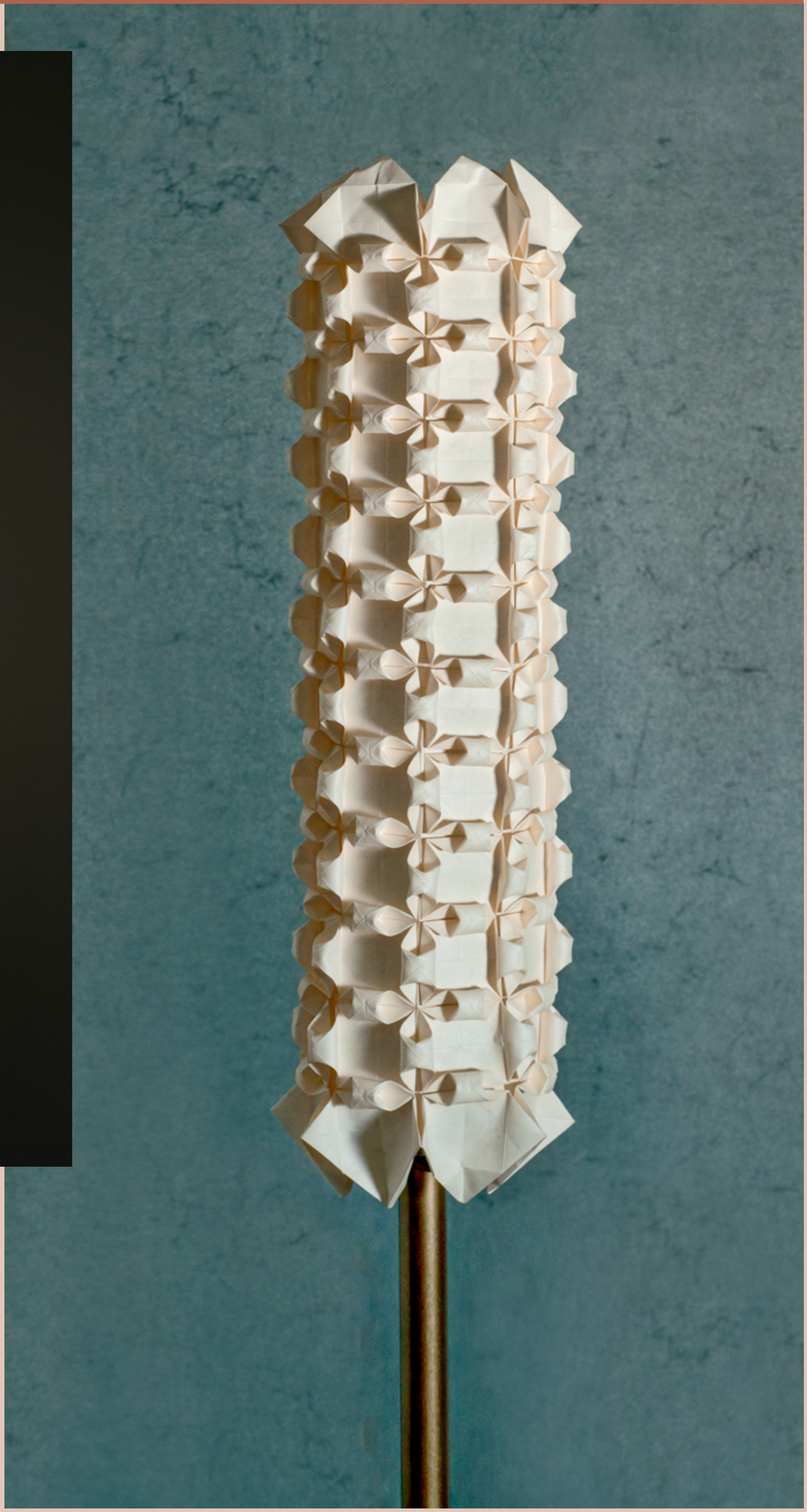
The final model will be eight squares wide.

Make all the precreases: fold all the mountain fold lines slanting in the same direction first, then fold all the creases slanting the other way. Now add all the valleys.

The collapse is done row by row, starting from the top left molecule. Extend the folds all the way to the edges of the paper and form the first molecule alone. To create the next molecule, make the mountain fold square first, and then sink the inner square.

Above and Beyond—A Floor Lamp

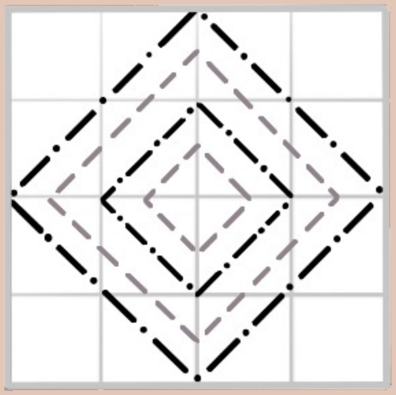




This lampshade is made from a sheet 8 molecules wide by 11 molecules high. To lock the sheet into a tube, the right column is aligned on top of the left column and they are collapsed together.

3 2 Double Red Flower

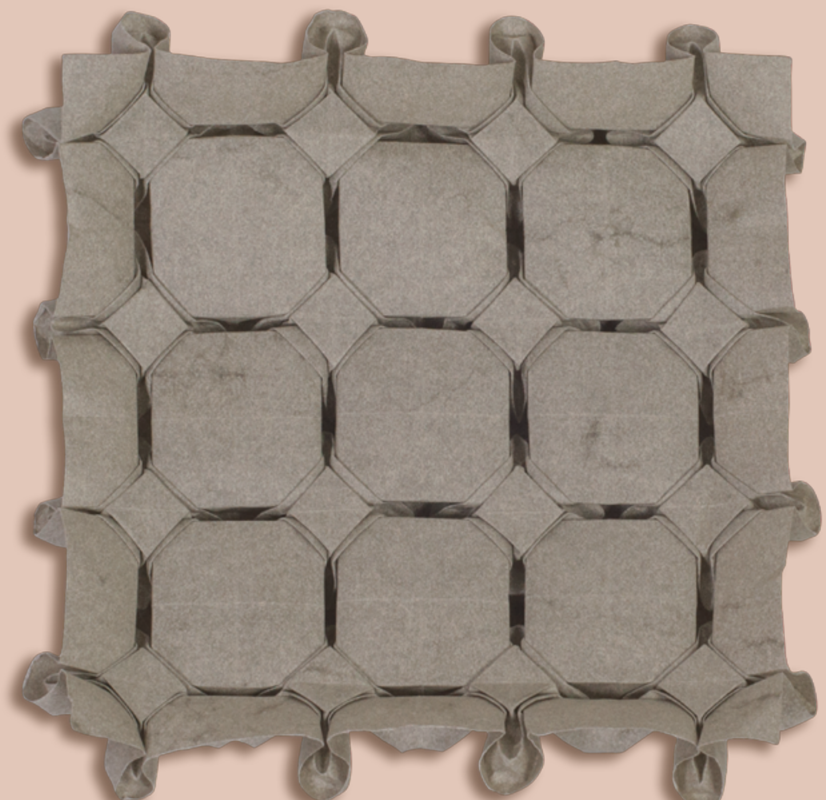
The Single Molecule



This molecule is the result of doubling the number of concentric squares from the Red Flower tessellation (page 74) to give more details to the petals. This is done by dividing the space between the tilted square to the center point by four, not two.

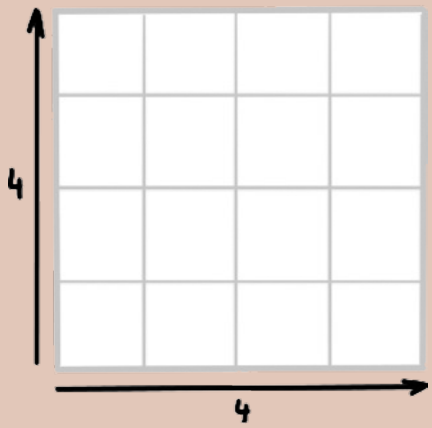
I highly recommend that you fold the Red Flower tessellation first for practice.

The shrinkage ratio is 2:1.

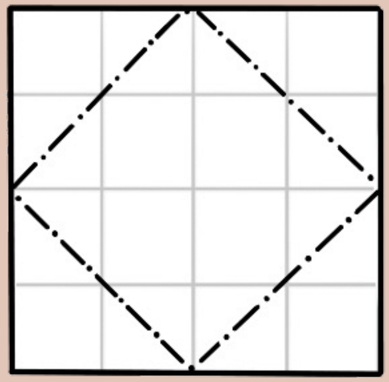


Above: recto view of a 4 by 4–molecule Double Red Flower tessellation.

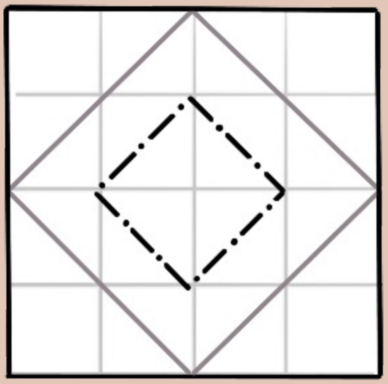
Verso view is to the right.



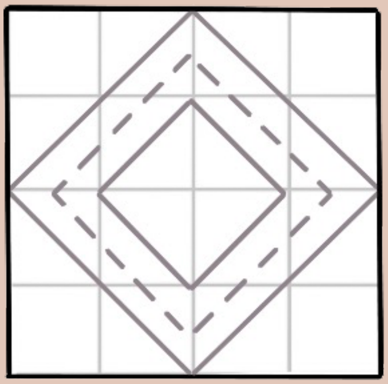
Start with a 4 by 4 grid. For this model there is no need for an extra row or column around the molecules.



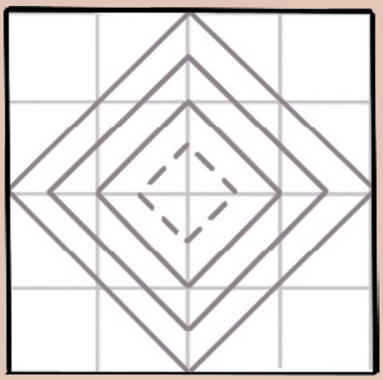
Mountain fold a tilted square, connecting all the midpoints of the grid.



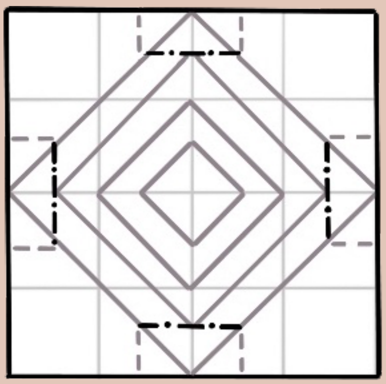
Add a smaller inner square, as mountains again.



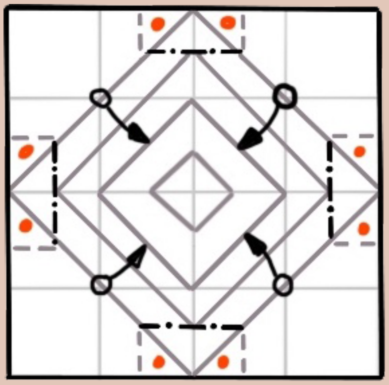
Now add the valleys. Pinch the outer mountain line and bring it to the inner one.



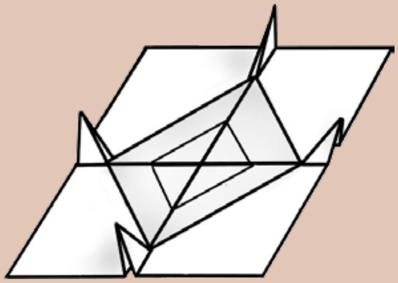
Now add the smallest square with valleys. Do it by pinching the inner mountain line and bringing it to the center.



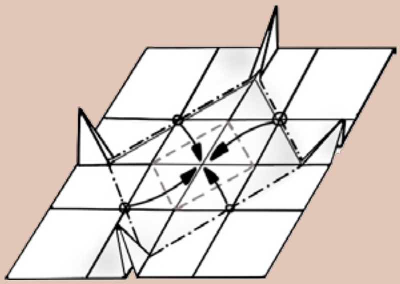
You can add these creases for an easier collapse (or you can skip this step). Note the orientation of the creases!



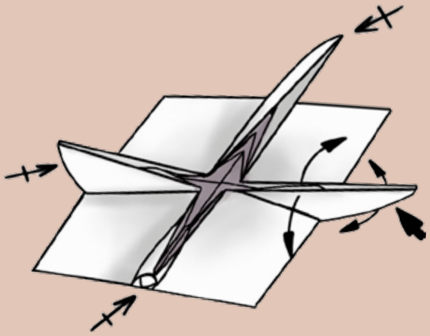
Start the collapse with the outer mountain and valley squares. Pinch the corners of the tilted square.

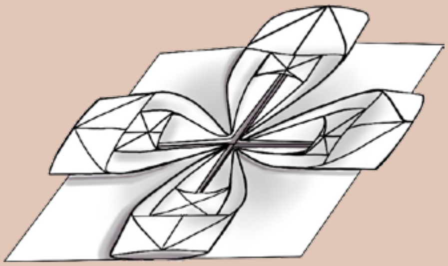


The result.



Now repeat the process on the inner squares.





This results in double-layered triangles.

Spread each petal, starting with the inner parts, using your thumb. Once the inner part is spread, it is easier to spread the outer one, starting from the highest point.

The result.

You may need to use a cotton swab to puff and round the inner parts from beneath.

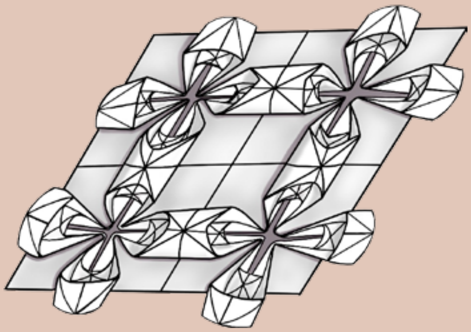
2 by 2 Molecules

Use a grid of 8 by 8. Add all precreases.

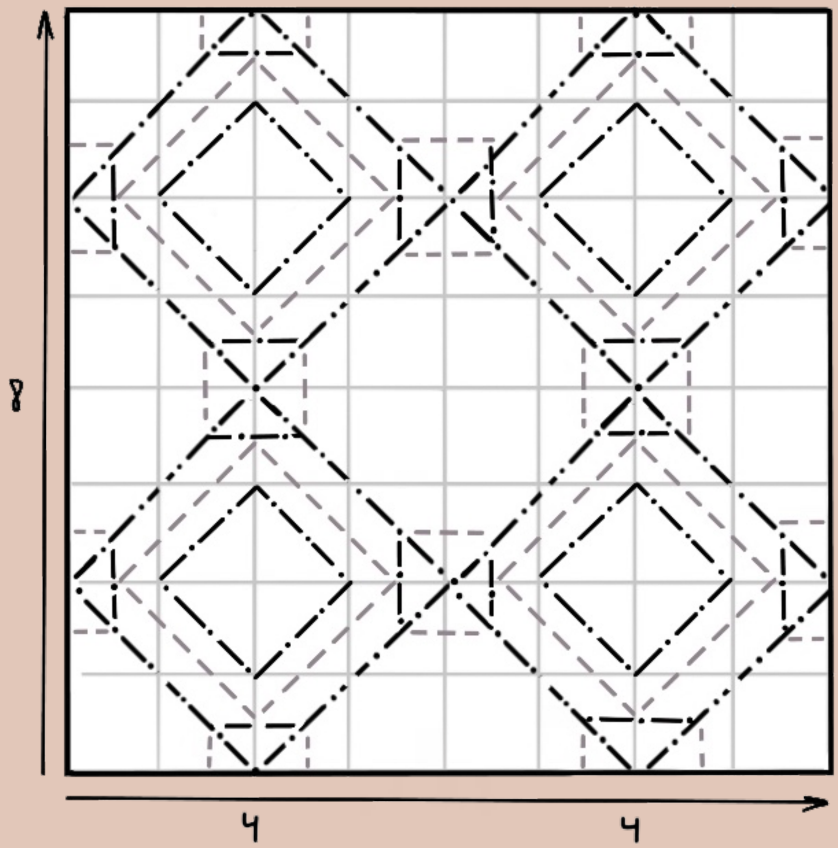
Note that the mountain lines of the bigger squares align with each other and can be creased with one stroke.

All the other squares must be made line by line; there is no shortcut for that.

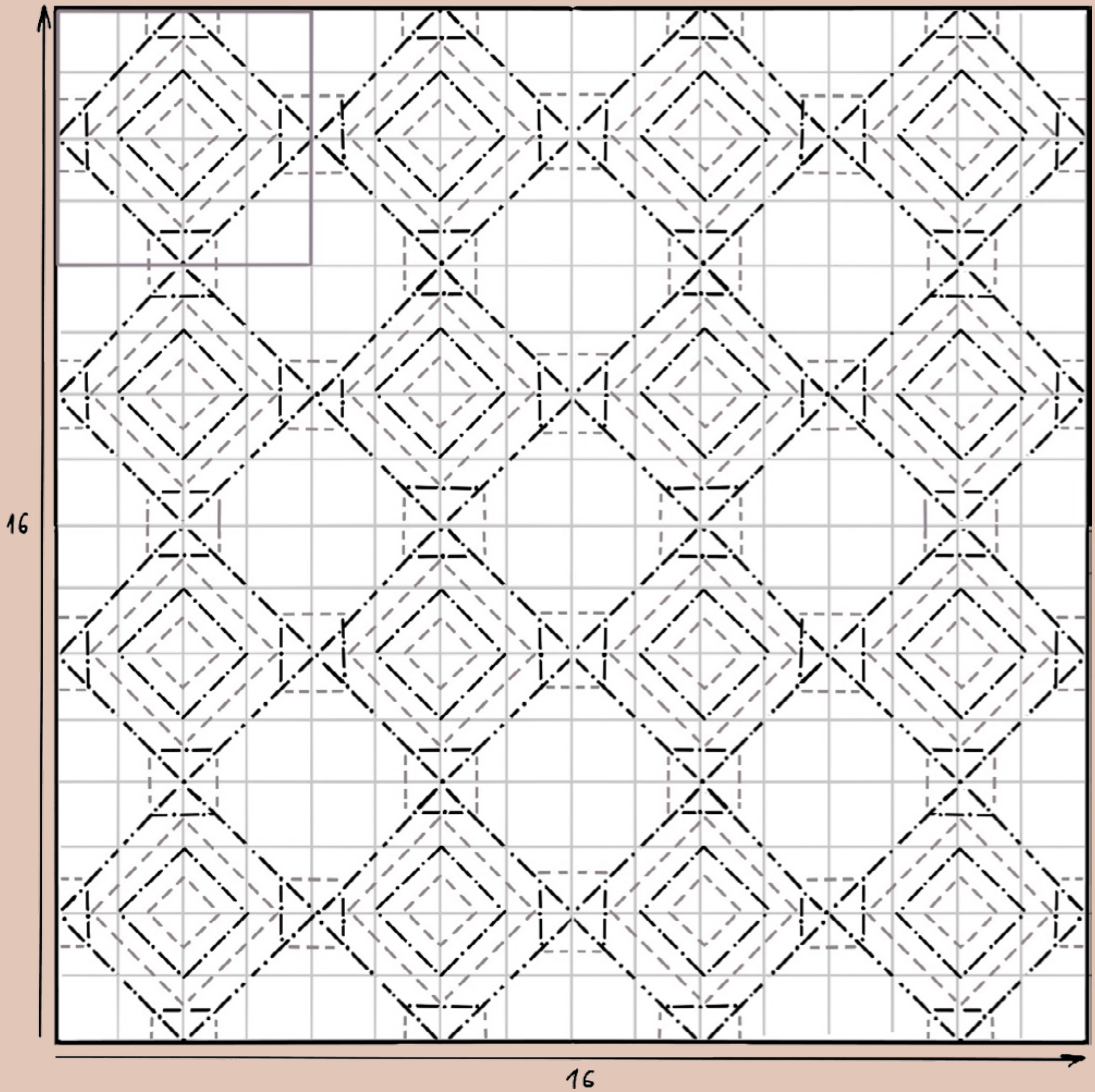
Collapse the project in two stages. First, collapse the bigger squares all over; only then should you complete the collapse with the inner squares.



Always puff the inner petals first. Use a cotton swab to puff the petals from beneath.



4 by 4 Molecules



Use a grid of 16 by 16: 4 x 4 = 16.

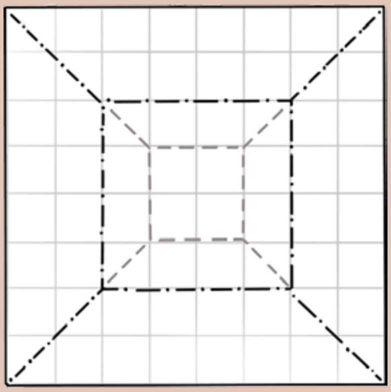
The final model will be eight squares wide.

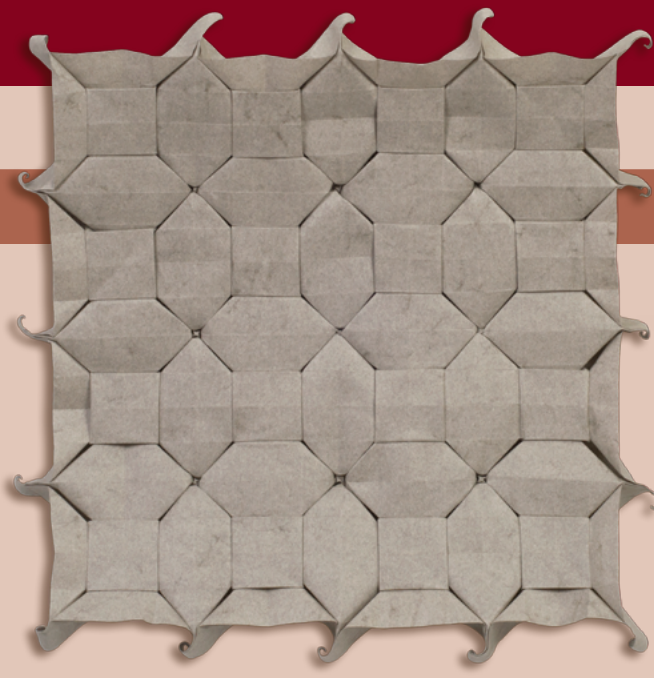
Add all precreases.

Follow the process presented in the 2 by 2 project, collapsing the bigger squares first and then the small ones.

3 3 Green Flower

The Single Molecule



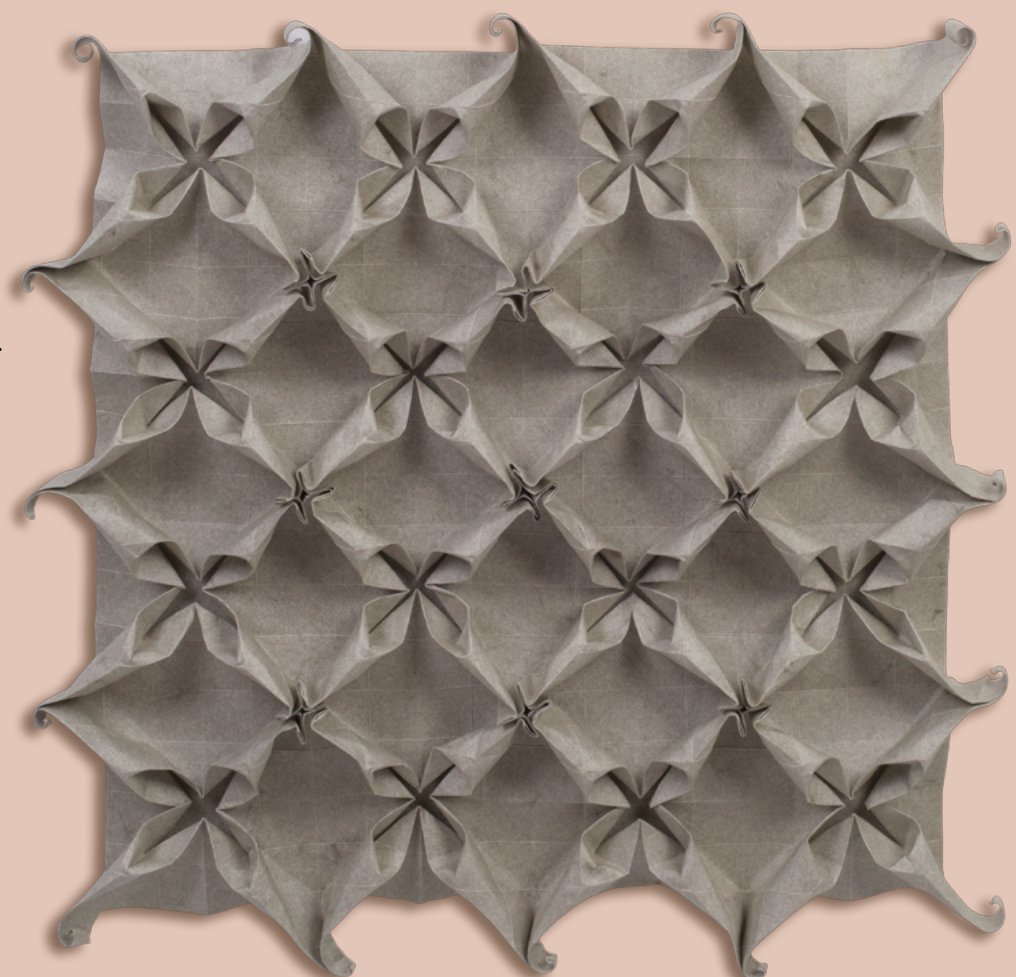


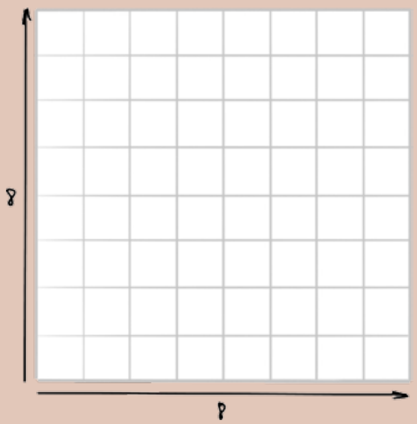
In this variation, the inner squares are not tilted. Instead, they are parallel with the molecule edges.

Bringing the midpoints of the outer square to the center is still part of the process.

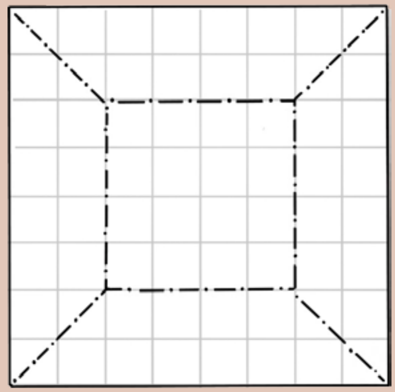
The shrinkage ratio is 2:1.

Top: recto view of a 4 by 4–molecule Green Flower tessellation. Verso view is below.

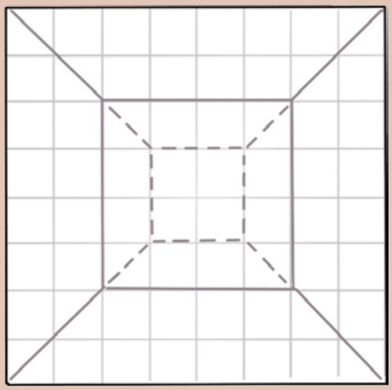




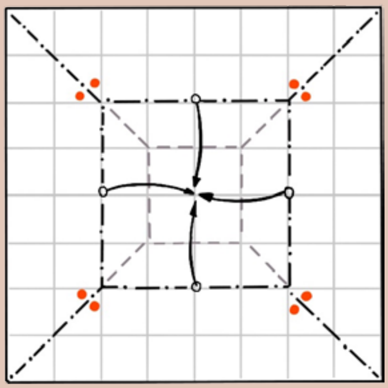
Start with an 8 by 8 grid. This molecule does not have an extra row or column.



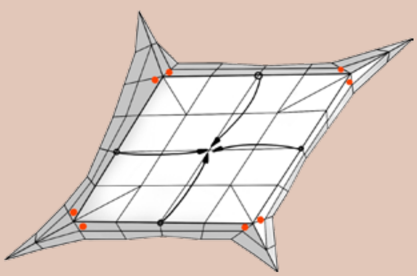
Mark a 4 by 4 square in the center with mountain creases. Add the diagonals as mountains, but do not fold them in the inner square.



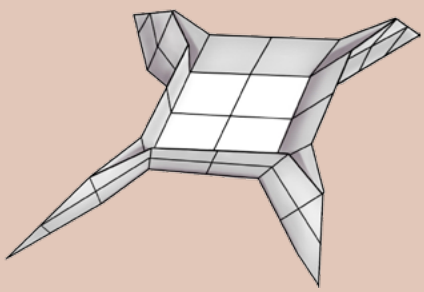
Add a 2 by 2 square in the center and mark the four short diagonals, all as valleys.



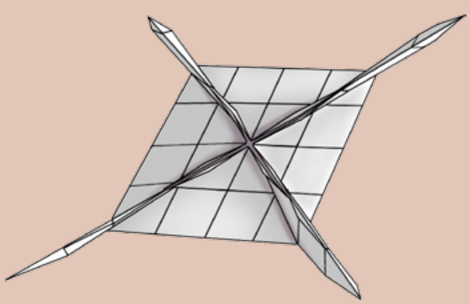
Start the collapse by bringing all the outer square midpoints to the center.



In process.

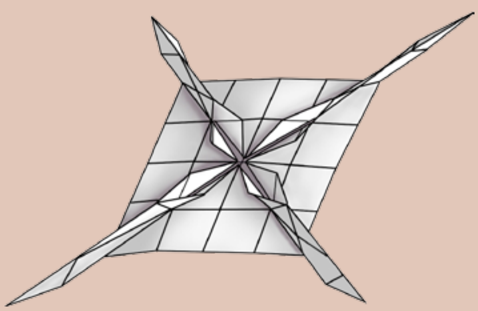


Almost there.



Fully collapsed.

Puff the petals by spreading the lips of the pockets open.



The complete molecule.

Note that the diagonals extend past the square base. This will require a special solution when we position two molecules next to each other.

2 by 2 Molecules

Use a grid of 16 by 16.

Add all precreases. Follow the diagonals, but skip the central 2 by 2 squares at the center of each molecule.

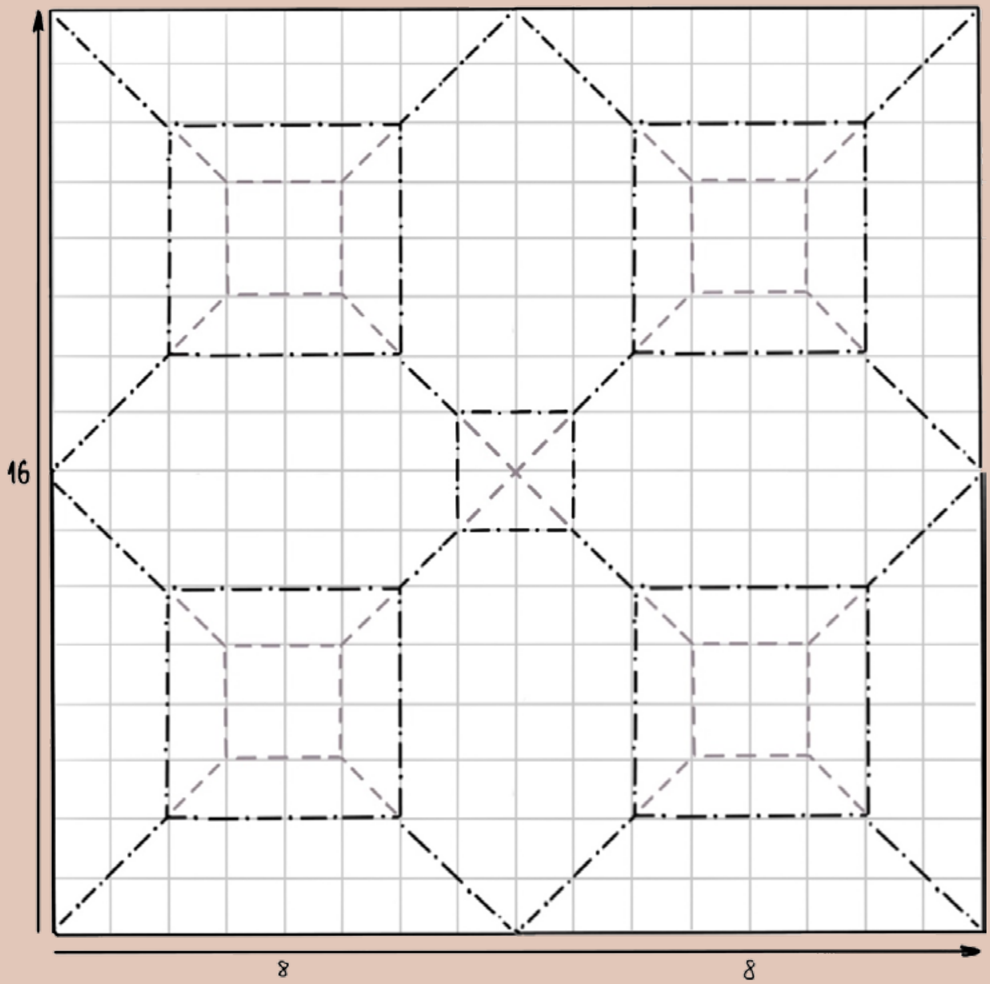
Start to collapse the top left molecule, and then complete a row.

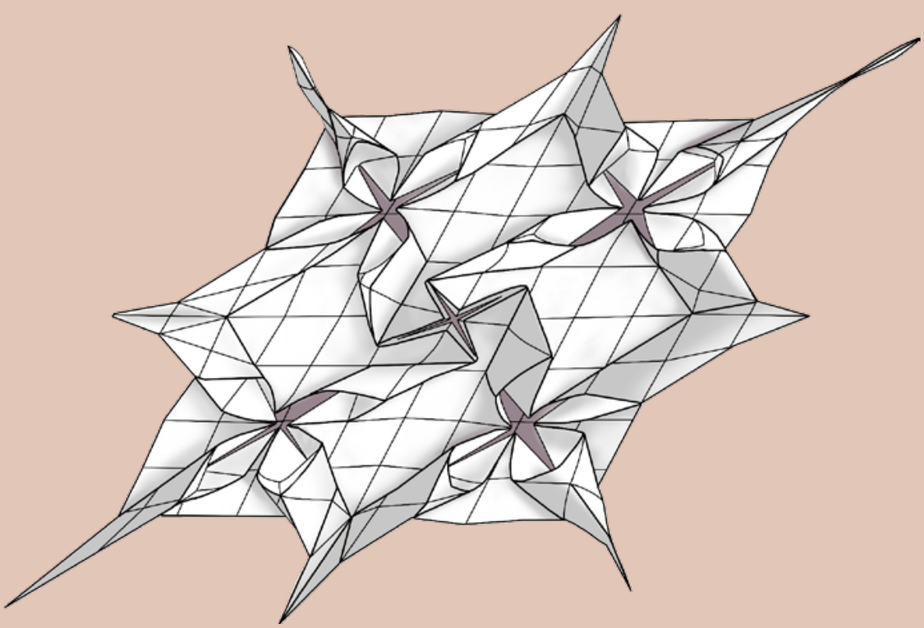
There will be a lot of tension in the paper while collapsing. The corners of the four molecules coincide in the center of the model, and it is impossible to keep the paper flat. To solve that, twist this area, as is shown in the isometric view below.

Do not give up!

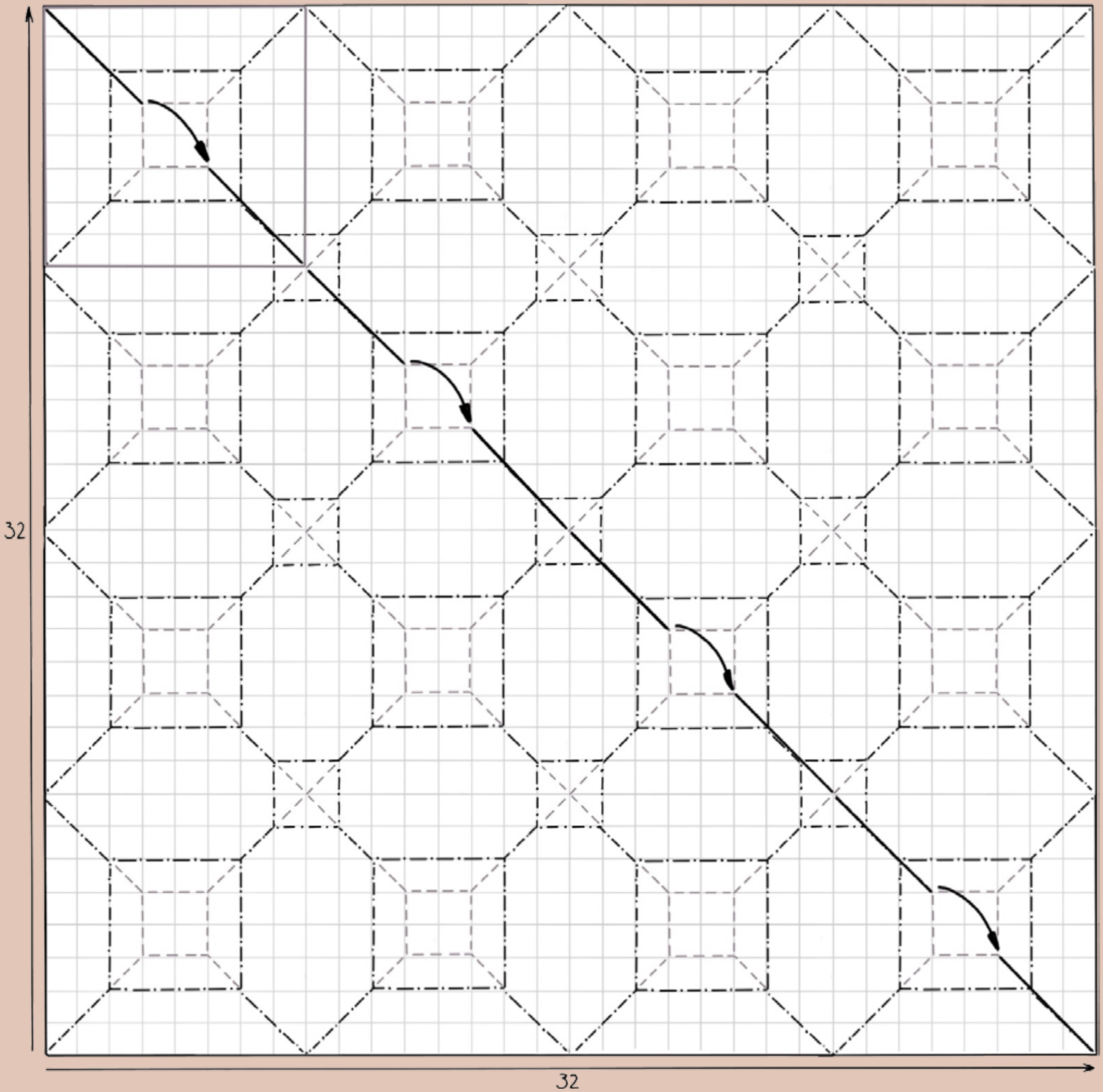
During the collapse, allow the molecules to remain open a little, to dial down the pressure.

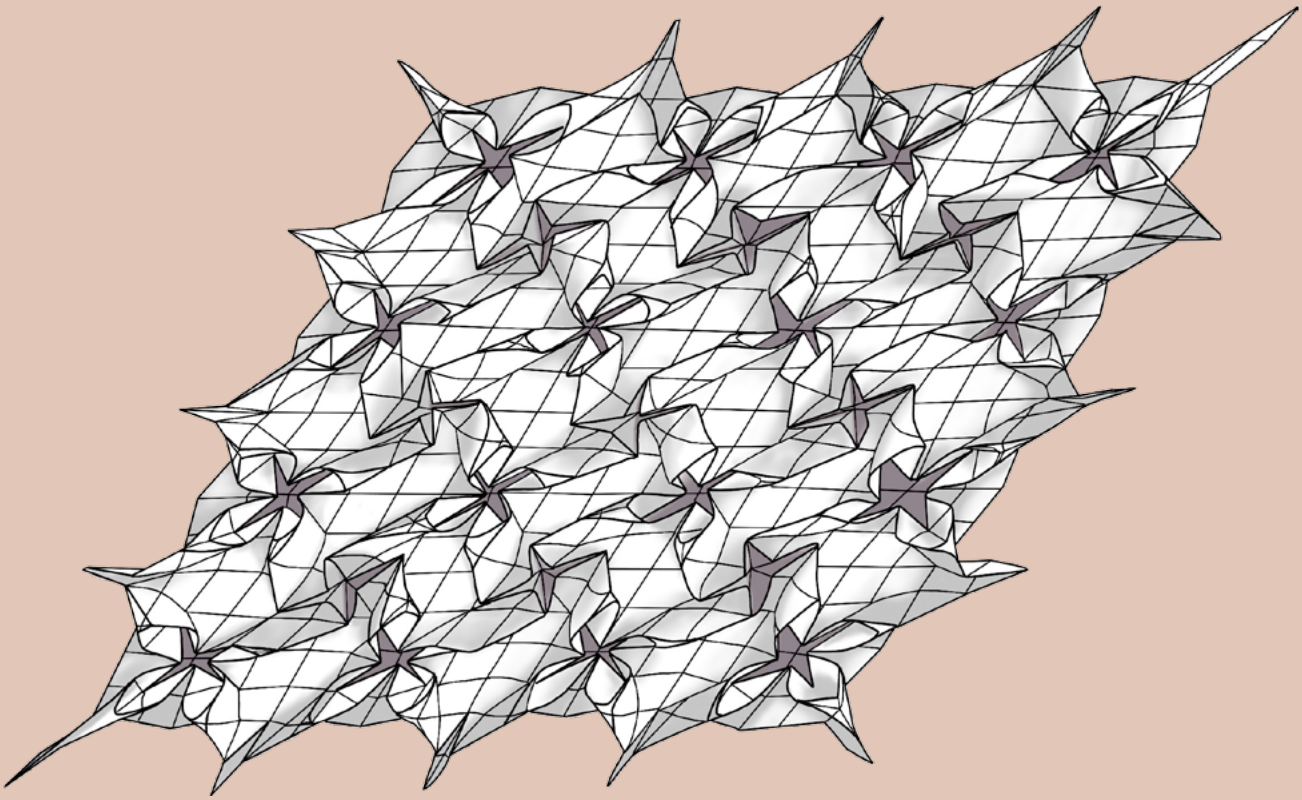
Note the water bomb in the center. This is the result of the diagonals extending beyond the 4 by 4 inner square of the molecule. In order to stack them in, you have to twist this part. You can also sink this part in the shape of a water bomb base.





4 by 4 Molecules





Use a grid of 32 by 32:
4 x 8 = 32.

The final result will be 16 squares wide.

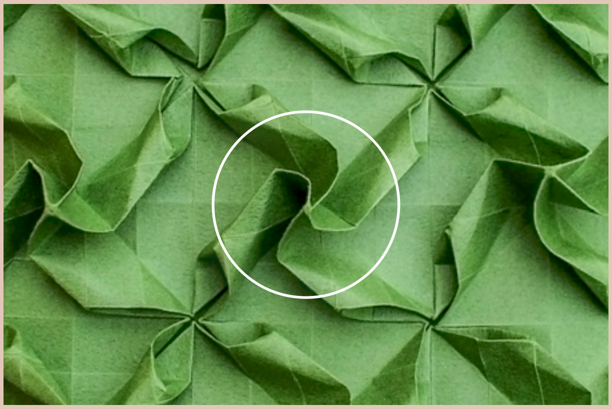
Add all precreases, following the diagonals of every molecule, but skip the inner 2 by 2 squares.

Do remember to reverse the orientation of the inner parts of the diagonals from mountains to valleys!

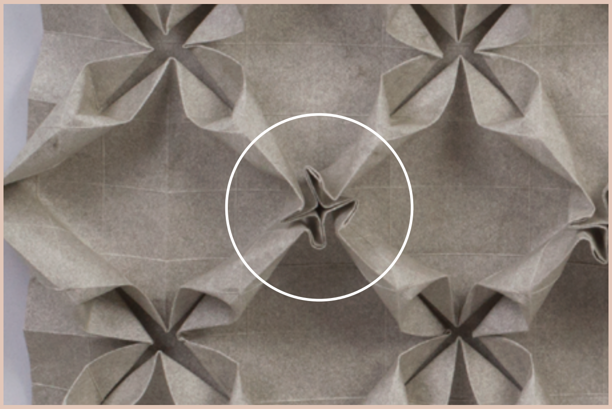
Collapse row by row, following the 2 by 2 project process. In this project there is a lot of tension between all the molecule corners, so work carefully.

You can choose to sink all the water bombs or not.

To puff the inner parts of the flower, use the same technique presented in the Red Flower tessellation (page 76). The lips of the opening tend to lie to one side. To get the flower petals to align with the diagonals, you can pinch the extra paper, as shown below right.



In the original version, the water bomb is left untouched.

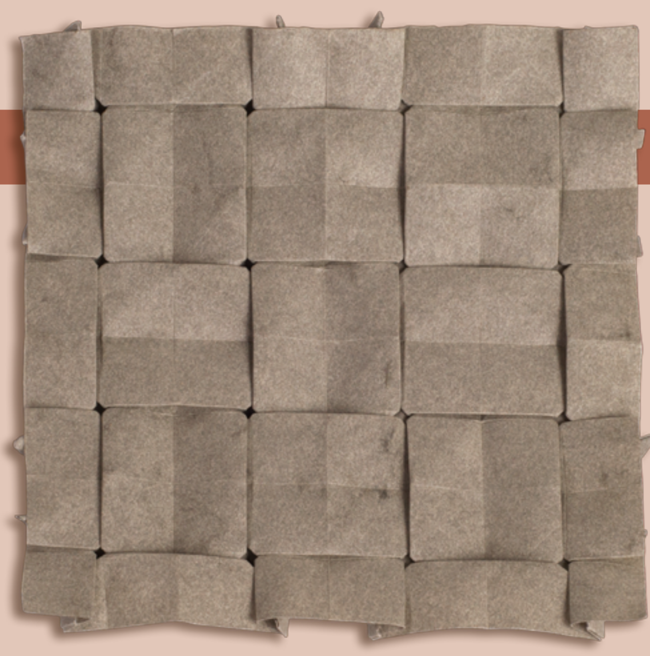


The extra paper is pinched shut for a cleaner look.

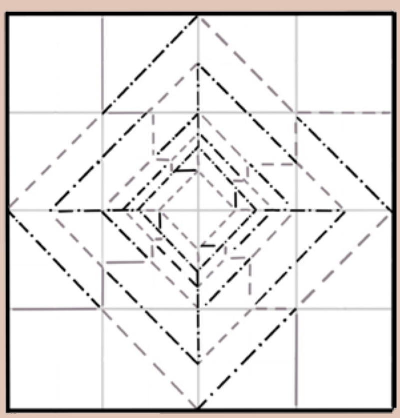
34 Bagan

Top: recto view of a 4 by 4–molecule Bagan tessellation.

Verso view is below.



The Single Molecule

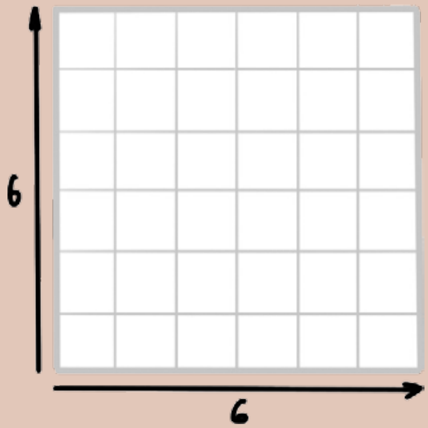


Out of the first step of the Red Flower tessellation (page 74) comes this multi-layered, pagoda-like construction.

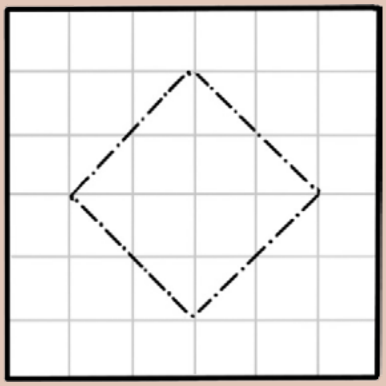
Tessellating it reminded me of Bagan, a city in Myanmar with thousands of pagodas.

Compared to the Red Flower molecule, there are more fold lines. You can choose to fold them in advance or make them during the collapse process. The collapse is easier with them, but they make the precrease stage much longer.

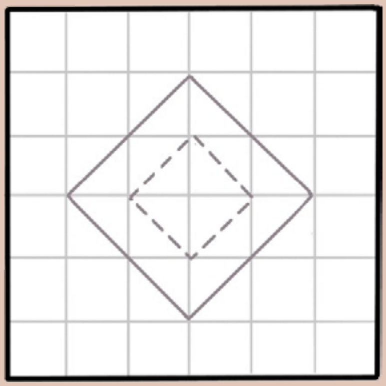
The shrinkage ratio is 2:1.



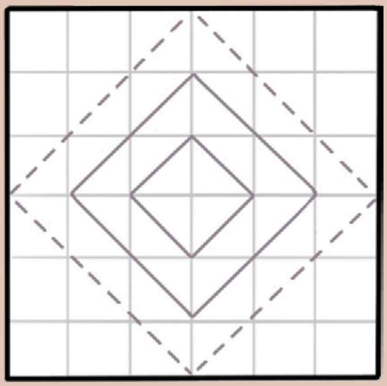
Start with a 6 by 6 grid. This includes an extra row and column that will help during the collapse.



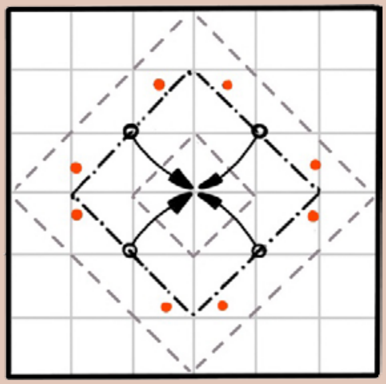
Add a tilted square with mountain folds.



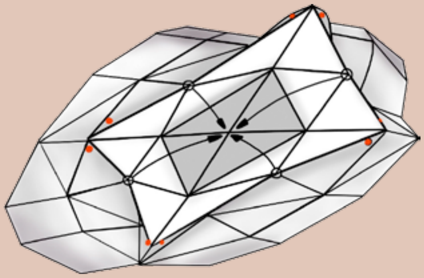
Add a smaller square with valley folds.



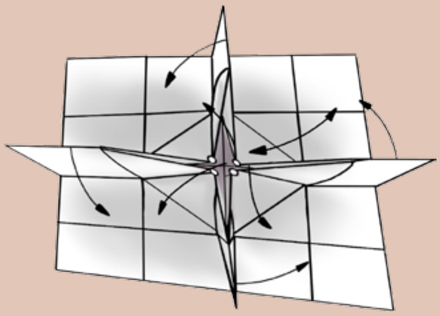
Add a bigger square, also out of valley folds.



Collapse the first stage as outlined in the Red Flower tessellation (page 75).

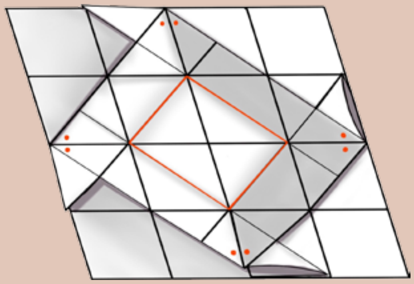


Collapse in process.

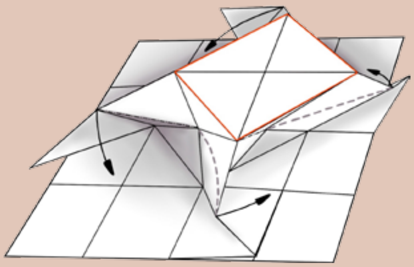


The first stage is fully collapsed.

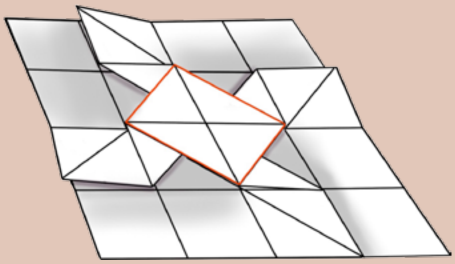
Pull out the four midpoints and spread out the square.



The result. Pinch the four corners (marked with red dots) of the upper square to form a cube. The red square will rise and form a cube shape. You can push it from below, if needed.

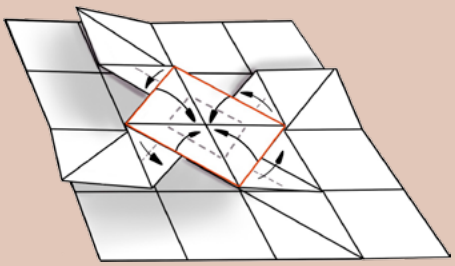


Now flatten the cube again, while relocating the marked flaps. Push the red square down.

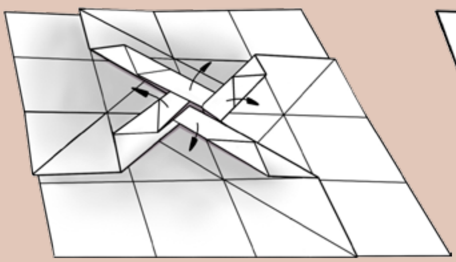


The result.

Note that the red square does NOT rotate during the collapse! It is just lower than before.

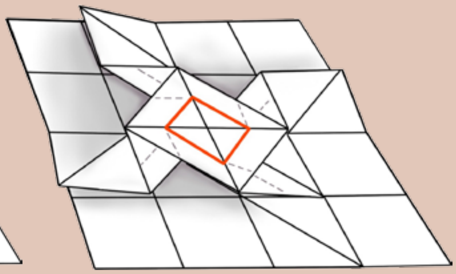


For the second level of the pagoda, fold edges to the center to form a smaller square. At the same time, narrow down the four flaps as shown.

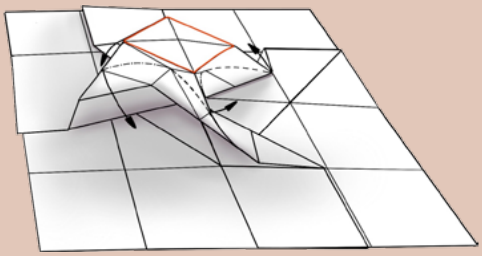


The result.

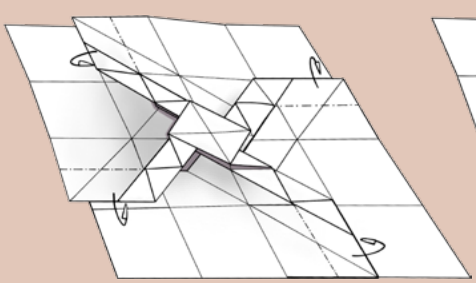
Unfold the flaps; they were precreases for the next step.



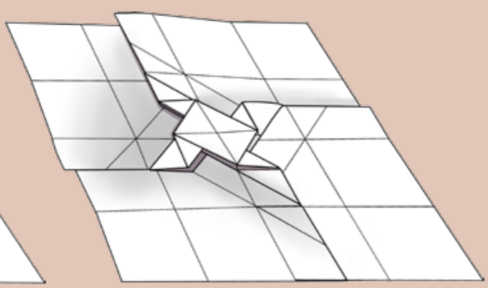
Repeat the process of pinching and raising the central square. You may need to use a long tool to push the paper up from the inside.



Flatten the square again, while relocating the flaps, as with the first level.

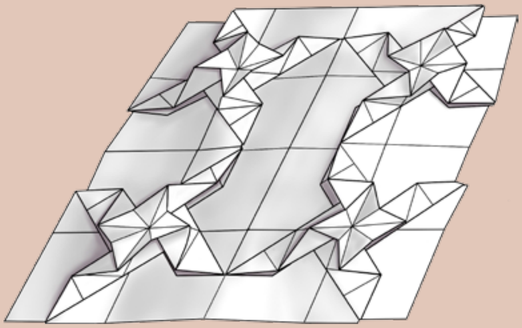


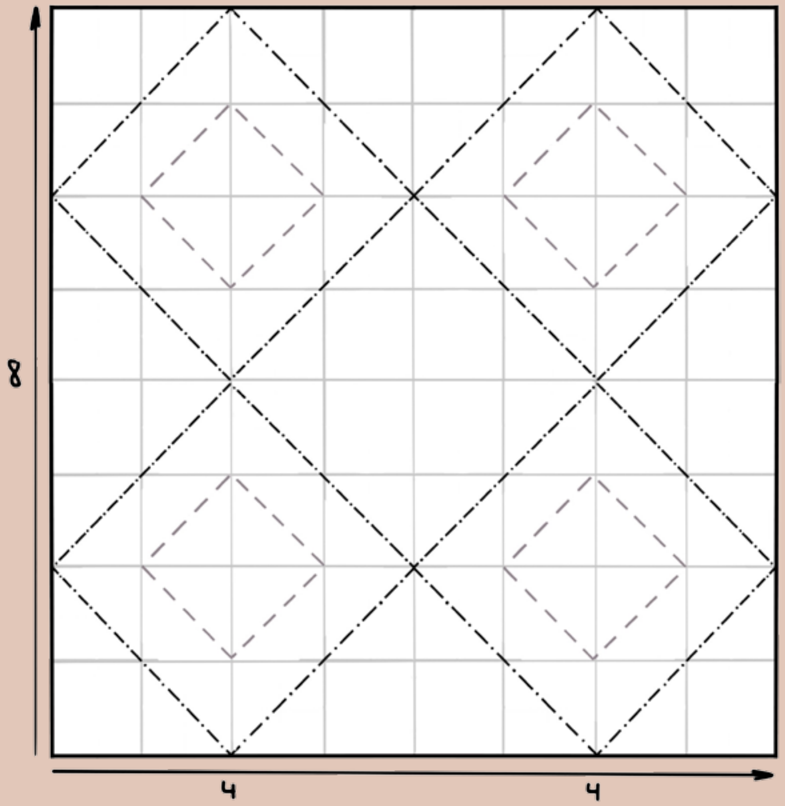
Mountain fold the four flaps as shown to hide them.



The finished molecule.

2 by 2 Molecules





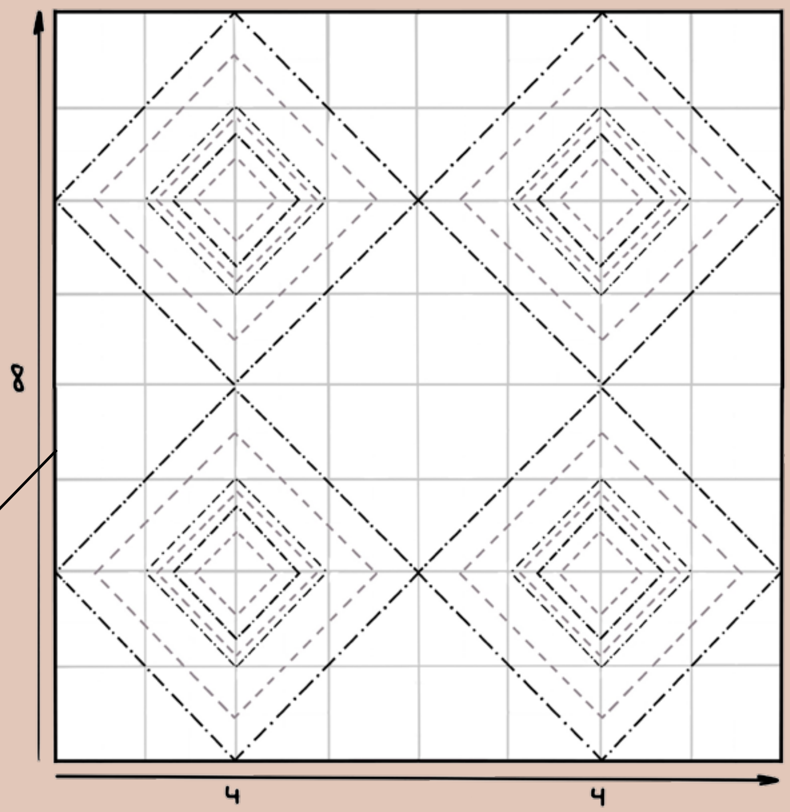
Use a grid of 8 by 8.

Although the single molecule was folded from a 6 by 6 grid, this project is tessellated without an extra column and row, affording a cleaner look.

Add all precreases, following the Red Flower tessellation process on page 76.

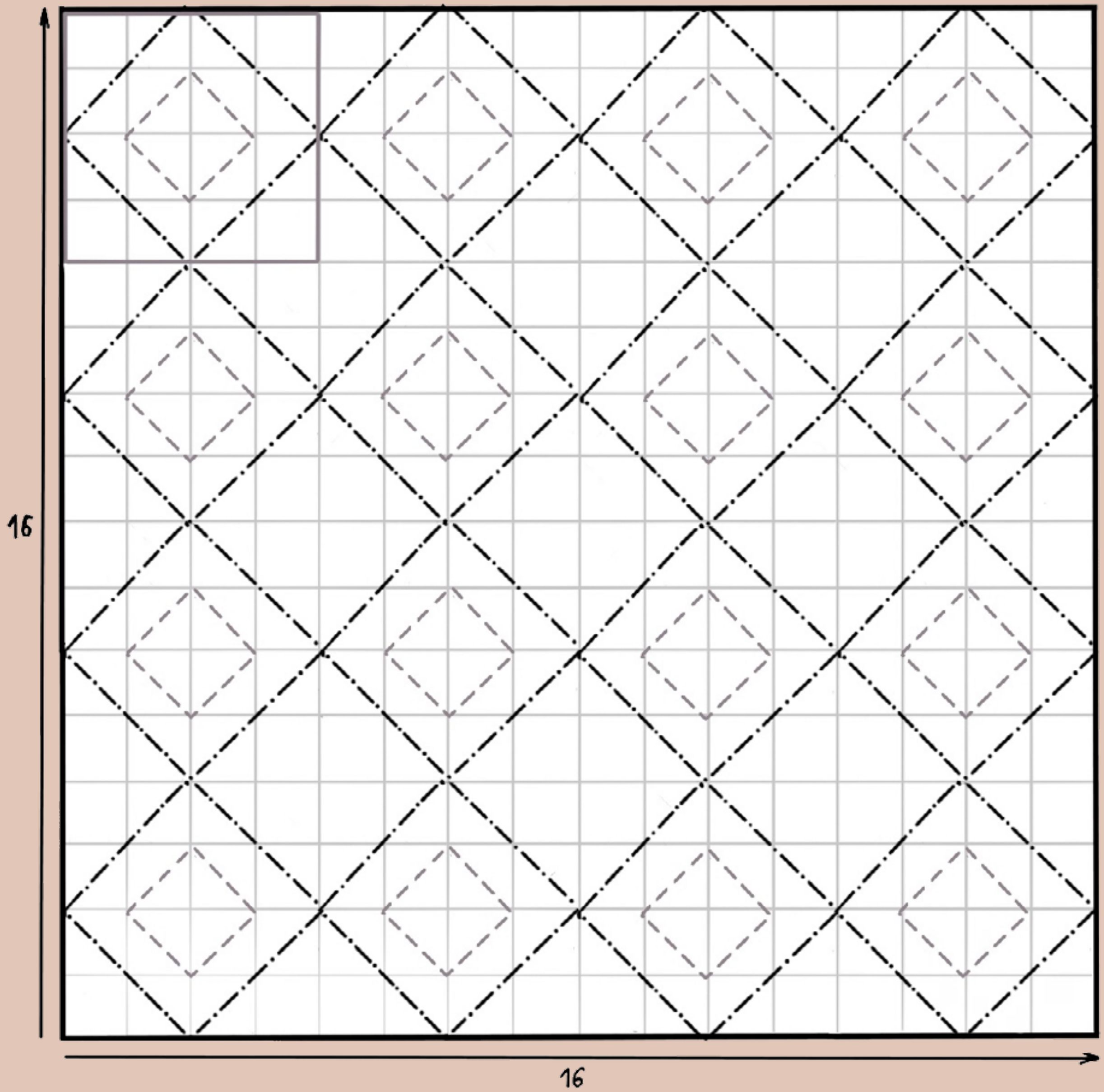
After the model is fully collapsed to the un-puffed stage of the Red Flower tessellation, you can work on each molecule separately, and turn it into a Bagan tessellation.

You have to fold this tessellation one level at a time. Complete all molecules to the same level before moving on to the next one.



If you want to make the collapse stage easier, follow this CP. The fold lines you have to add to narrow the flaps during the collapse are done beforehand!

4 by 4 Molecules



Use a grid of 16 by 16:
4 x 4 = 16.

The final result will be eight squares wide.

Add all precreases; there are shortcuts for the first level of mountain lines only.

You can add all the other creases not shown here beforehand, as with the 2 by 2 project on the previous page.

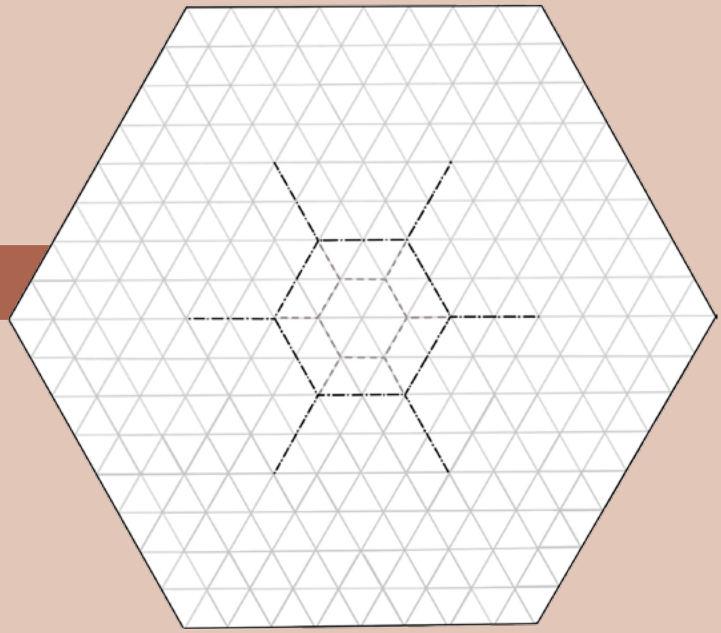
Collapse row by row.

You have to complete all the molecules to the same level before working on the next one.

3 5 Hexa Flower

Right: recto view of the Hexa Flower.

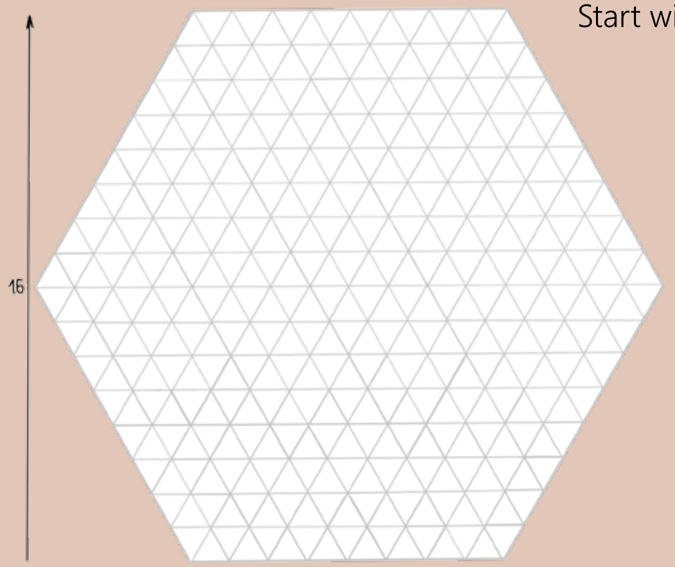
Verso view is below.



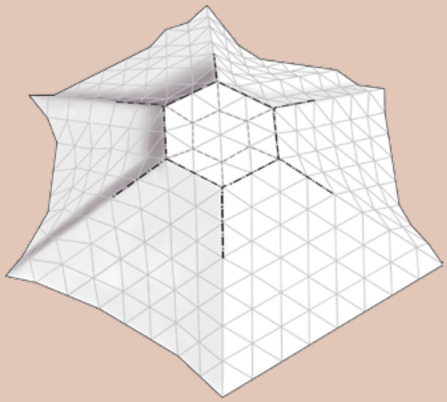
The Single Molecule

For hexagon grid projects, we start in the center. Following the same approach covered in the square grid projects, we bring the midpoints of the hexagon sides to the center point. This creates the center flower around which more flowers are added.

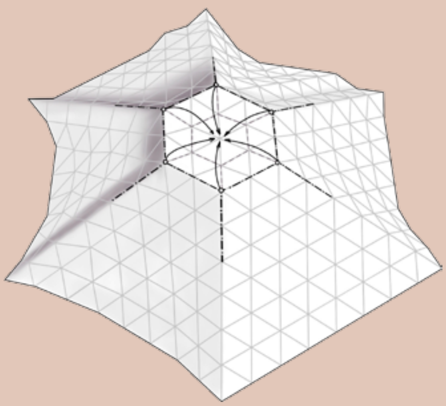
16 Grid Project

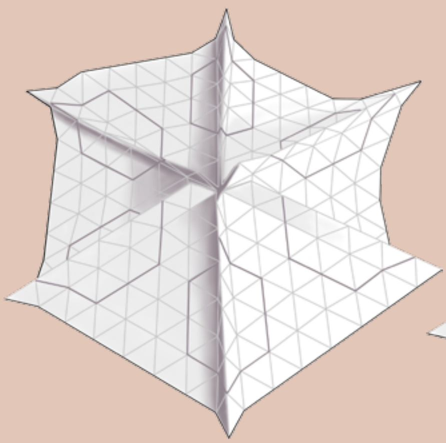


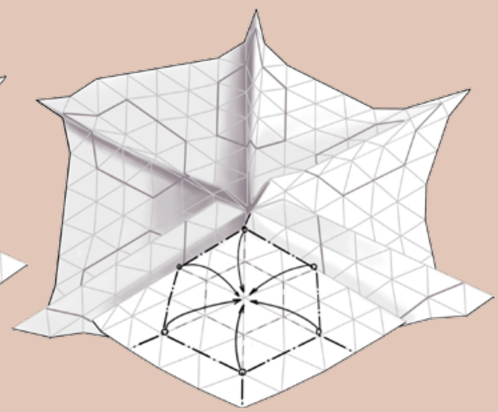
Start with a grid of 16 by 16 by 16.



Mountain fold the second inner hexagon and all the fold lines going to the corners of the sheet.



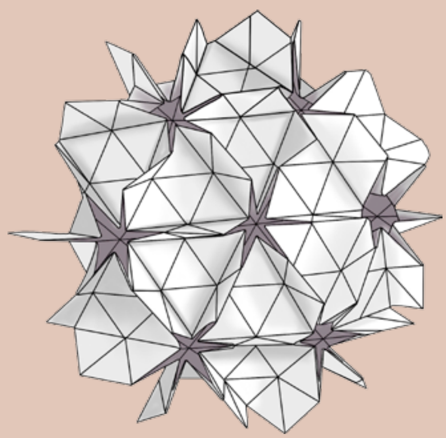




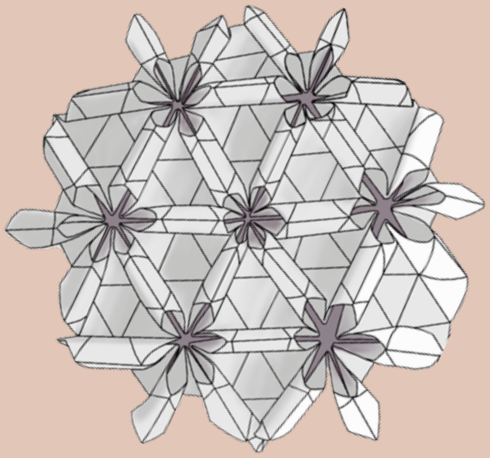
Bring all the corners of the inner hexagon to meet in the center, following the process of the Red Flower molecule, shown on page 75.

The first flower is collapsed.

Flatten the paper around the next hexagons and repeat the process. Continue in this fashion until all the flowers are complete.

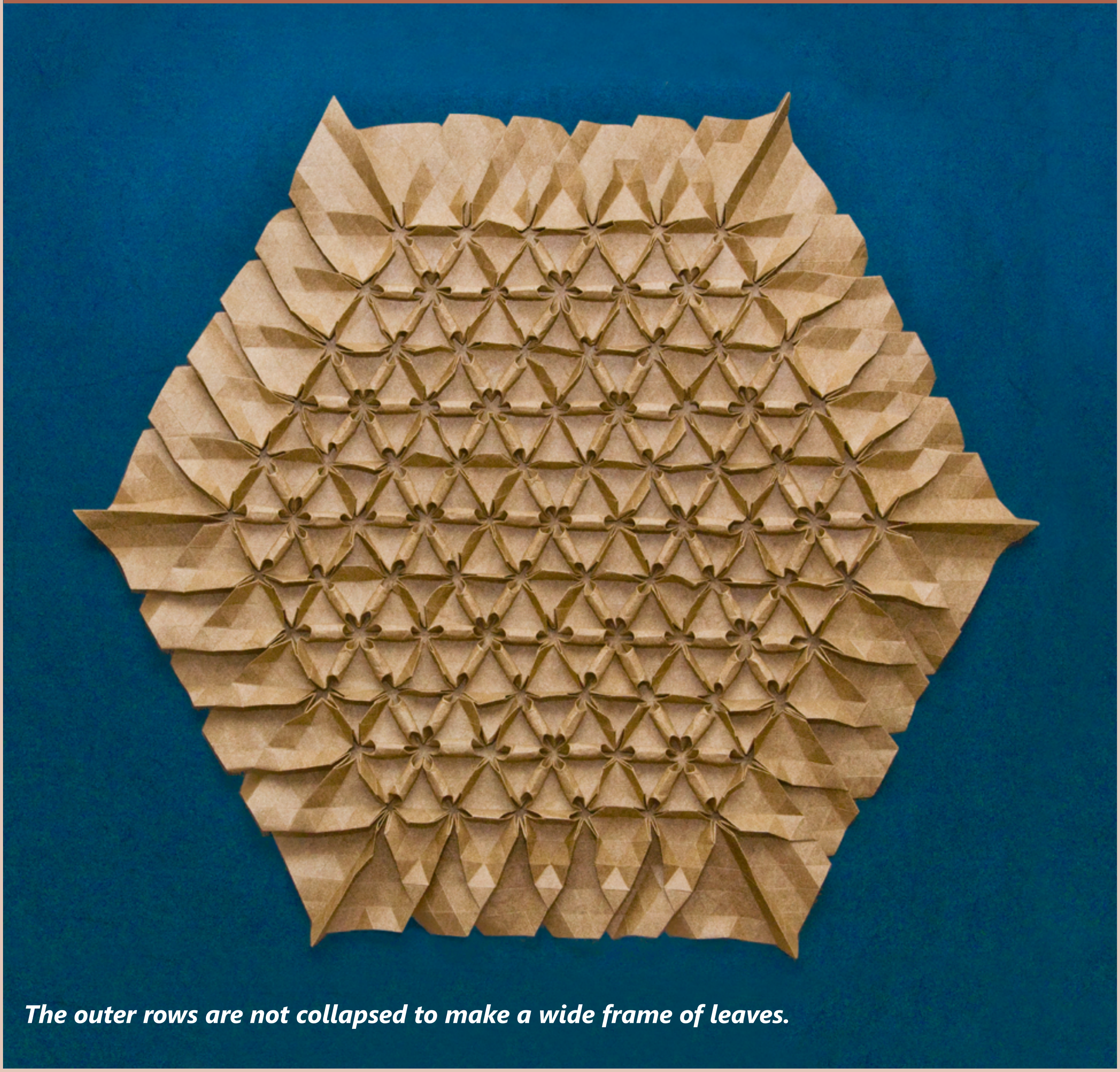


The result is seven flowers.



You can puff every petal (or just some) using the same process from the Red Flower tessellation (page 76), squashing the lips slightly with your fingers.

Above and Beyond—A 64 Grid Project



The outer rows are not collapsed to make a wide frame of leaves.